Parts and differences

Stephen Yablo

Abstract   Part/whole is said in many ways: the leg is part of the table, the subset is part of the set, rectangularity is part of squareness, and so on. Do the various flavors of part/whole have anything in common? They may be partial orders, but so are lots of non-mereological relations. I propose an "upward difference transmission" principle: x is part of y if and only if x cannot change in specified respects while y stays the same in those respects.

1 An unselective relation

If I tell you that x is a life coach, you know that x is a person. To divide y evenly, x should probably be a number. x and y had better *not* be numbers if x is closer to the North Pole than y.

To learn that x is or has a *part*, however, tells you nothing about the sort of thing it might be, considered in itself. Philosophers have discovered some strange entities over the years, but nothing so ontologically outré as not to stand in mereological relations. So, for instance,

the leg is part of the table
"sky" is part of "skyscraper"
Saturday is part of the weekend
Maine is part of New England
The 14th Amendment is part of the Constitution

S. Yablo (✉)
Department of Linguistics and Philosophy, MIT, Cambridge, MA, USA
e-mail: yablo@mit.edu

Of course, we should be able to make a longer list than this, if the relation is truly unselective. Inclusion should make sense in connection with any category of object you like. One notable attempt to extend the list was Lewis's in *Parts of Classes.*[1] The set of dogs is part of the set of mammals; subsets more generally are part of their sets. Lewis was inventive, too, with events:

> John says "Hello." He says it rather too loudly… Arguably there is one event that occurs which is essentially a saying "Hello" and only accidentally loud [and] a second event that is essentially a saying-"Hello"-loudly (Lewis 1986, 255)

"There is a clear sense," as he says, "in which the [first] event is part of the second" (256). To be a triangle is part of being a right-angle triangle. The firebombing of Dresden was part of WW II, and part, in another sense—the same as that in which John's saying "Hello" is part of his saying it loudly—of Dresden being firebombed on those particular days. Training in CPR is part of what is required for being a lifeguard. "Part of the trick to making poached eggs is to put the egg in a fine mesh sieve before lowering it into the water." "Part of why nobody trusts him any more is that exact type of behavior." And so on. It seems that we have at least some ability to project our understanding of part/whole into new territory.

2 Parts as such

Is there some common element or theme that guides us in these projections? One would like to think that part/whole is the same relation, or the same *type* of relation, in all its incarnations. The leg and the table are carrying on in the same sort of way as Saturday and the weekend, "sky" and "skyscraper," Maine and New England, and so on. A first thought might be

$$\text{Part/whole is transitive, reflexive, and antisymmetric: a partial order.} \quad (1)$$

Partial-orderhood is too weak, though, to be the feature common to all flavors of part/whole, thanks to which they *are* flavors of part/whole; for "most" partial orders have nothing mereological about them. Some may also think it too strong. Certainly we can keep the proper relation in mind even while suspending judgment on features like transitivity. Take these in turn.

Coming-later-in-the-week-than—the relation day x bears to day y if x is Tuesday and y is Monday or…or x is Sunday and y is Saturday—is a partial order, thinking of the week as extending from Monday to Sunday. Saturday comes later in the week than Friday, yet it is not part of Friday. Why not? One might look here to mereology's other axioms, beyond those defining a partial order. The one usually mentioned next is *Supplementation*:

> y is a proper part of x only if a z exists that "makes up the difference" between them — meaning, it is disjoint from y and sums with y to form x.

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[1] Lewis (1991). See also Yablo (1992).

Certainly it is hard to think of an z that counts intuitively as what Friday adds to Saturday, or as what is left over when Saturday is removed from Friday. This leads to a second proposal[2]

$$\text{Part/whole relations are partial orders with difference-makers.} \qquad (2)$$

A reflexive, antisymmetric, transitive relation $\leq$ relates parts to wholes, according to (2), iff for all $y < x$, a $z \leq x$ exists with two properties:

(i) z is disjoint from y; there is no $u \leq z$ such that $u \leq y$, and
(ii) $y + z = x$; whatever overlaps with y or z overlaps with x and vice versa.

Later-in-the-week-than does not satisfy (2) as we intuitively understand it. Doesn't that suggest that Supplementation captures the missing element? No, it suggests that we *understand* Supplementation in such a way that nothing counts as what Friday adds to Saturday. It's not that the axiom *itself* is so demanding; rather our instinctive construal of it goes beyond what the axiom strictly requires. Unintended models are not hard to devise. We can code finite sets of natural numbers as follows: $n_1, n_2, \ldots, n_k$ (ordered from smallest to largest) is represented by

$$p_1^{n_1} \times p_2^{n_2} \ldots \times p_k^{n_k}$$

where p_k is the kth prime. One natural number is "port" of another if and only if the set coded by the first is a subset of the set coded by the second (numbers that don't code any set are stipulated to be "port" of themselves). So, for instance, {4} is a subset of {4, 5}, so the number coding {4}, namely 2^4 or 16, is port of the number coding {4, 5}, namely $2^4 \times 3^5 = 16 \times 243 = 3,888$.

If the *sets* form a partial order with difference-makers under the subset relation, then the *numbers* form a partial order with difference-makers under the port relation. Port is antisymmetric, because if n is port of m and vice versa, then the corresponding sets are subsets of each other, and hence identical; identical sets have the same code, so $n = m$ as antisymmetry requires. And so on. Supplementation is satisfied because 32, for instance, a proper port of 3,888, makes up the difference between 16 and 3,888. Port despite this is not a genuine part/whole relation, I take it. Our sense of what we want in a part is not exhausted by the structural requirements imposed by the axioms.

Going in the other direction, is it clear that a genuine part/whole relation has to satisfy the stated conditions?[3] Prima facie counterexamples have been given to all of them. Against transitivity: the roots are part of the tree, the tree is part of the landscape, but the roots are not part of the landscape. Against antisymmetry: the universe is part of the proposition *The universe is large*; that proposition is part of the universe; but the universe is not identical to the proposition. Against

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[2] Residuated partial orders, we might call them (a residuated lattice is a partially ordered set with operations $\wedge$ and $\leftarrow$ related roughly as intersection and relative complementation. The existence of difference-makers corresponds to closure under the second operation).

[3] I am not taking a stand on this either way. The thought was only that we have must have some *other* way of thinking of part/whole, that lets us maintain our focus on it while questioning (e.g) transitivity.

supplementation: let the domain be all regions of space with positive volume or measure. A closed sphere of radius 1 properly includes its interior. But its interior has the same volume as the sphere. A difference, if there were to be one, would not take up any space, by additivity of measure. But then there can't *be* a difference.

3 Subject matter(s)

Not much progress has been made, to this point, on the problem of finding a thread running through part/whole in all its incarnations. Now I propose to make matters worse by looking at some further flavors of part/whole that seem even harder to bring into the fold. The first is part/whole as a relation on subject matters. Let me approach it in a roundabout way, via the notion of a "non sequitur."

Non sequiturs are usually understood, especially in academe, as fallacious arguments, but one sees occasionally a broader characterization. "In everyday speech," according to Wikipedia ,

> a non sequitur is a statement in which the final part is totally unrelated to the first part… It can also refer to a response that is totally unrelated to the original statement or question.

As examples they give, "Life is life and fun is fun, but it's all so quiet when the goldfish die,"[4] and this dialogue:

> Mary: I wonder how Mrs. Knowles' next-door neighbour is doing.

> Jim: Did you hear that the convenience store got robbed? They got away with a small fortune

One senses here a kind of topical disconnect, a sudden left turn in what the statements are about. I don't know quite what the subject matter of *Life is life and fun is fun* is, but it would seem orthogonal to that of *It's all so quiet when the goldfish die*.

Orthogonality and disconnect, considered as relations on subject matters, are interesting in their own right, regardless of any application to non-sequitur-hood. A theory of these relations has been developed by David Lewis.[5] The theory leads where we wanted to go, for the relation of disconnect is defined in terms of part.[6] It's that relation of part to whole that we're really after, of course, given the aims of this paper. Let's first see what Lewis has to say about the relata.

A subject matter, whatever else it may do, determines a function from worlds w to propositions stating how matters stand in w where it—the subject matter in question—is concerned. **The number of stars**, to use Lewis's example, maps worlds with equally many stars to the proposition that there are that many stars. Thinking of propositions as sets of worlds, we're talking about a function from

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[4] *West with the Night*, Beryl Markham.

[5] Lewis (1988a, b).

[6] See below; $\mathbf{m}$ is connected to $\mathbf{m}'$ iff they have a non-trivial part in common.

0 stars	1 star	2 stars	3 stars	4 stars	5 stars	...

Fig. 1 The subject matter **the number of stars**

worlds w to collections of sets of worlds—where, since the ways a world is **m**-wise are propositions true in that world, each set in the collection has w as a member. **m** will in the simplest case be a partition of logical space, with each world being mapped to its cell in the partition[7]

Two subject matters are *orthogonal*, for Lewis, if each cell of the first overlaps each cell of the second—as we see, for instance, with **the number of stars** and **the number of comets**. How matters stand where the one is concerned is logically independent of how they stand where the other is concerned. They are *disconnected* if they have no non-trivial parts in common. It remains to say what a part is.

The more inclusive subject matter, Lewis says, is the one it is easier for worlds to disagree on. **The nineteenth century** properly includes **the 1890's**, for instance, because worlds whose 1890s run parallel may yet have different things going on in their 19th centuries; but not vice versa. What this comes to in the case of partitions is that **m** refines **n**: its cells subdivide the cells of **n**.[8] A more general formulation (that does not require **m** and **n** to be equivalence relations, or to agree in their domains) is

1 **m** includes **n** iff

each **m**-cell lies within an **n**-cell, and
each **n**-cell includes an **m**-cell

Every way for things to be **m**-wise implies, in other words, a way for them to be **n**-wise, and each way for them to be **n**-wise is implied by a way for them to be **m**-wise.[9] **The number of stars** includes **whether there are over a billion stars** because

(i)   each way for things to be **number-of-stars**-wise—**There are exactly a hundred stars**, e.g.—entails a way for them to be **whether-there-are-over-a-billion-stars**-wise—**No, there are not**, in this case
(ii) both ways for things to be **whether-there-are-over-a-billion-stars**-wise are implied by ways for things to be **number-of-stars**-wise—**Yes** by there being a trillion stars and **No** by there being just a hundred.

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[7]   Lewis (1988b). (Fig. 1).

[8]   **m** and **n** are disconnected, then, insofar as there is no non-trivial **p** whose cells are subdivided both by the cells of **m** and by those of **n**. This will be the case if disjunctions of ways for things to be **m**-wise can never agree with disjunctions of ways for things to be **n**-wise, unless both are the set of all worlds whatsoever. Yablo (2014) shows how orthogonality and disconnectedness can come apart. See also Humberstone (2000).

[9]   I assume that each **n**-cell intersects at least one **m**-cell.

The number of stars doesn't include **the number of stars and planets**, because the stars could be more our less numerous for a given number of stars and planets.

4 Content parts

Armed with this account of subject matter inclusion, let us now try to push further into logical territory. Logicians are apt to say that a statement A's implications B are not new, but already "contained" in it. This ignores an important distinction.[10] Sometimes an implication "follows from" A, in the sense of being a downstream consequence. Other implications "precede" A; they are pre-required for its truth. Conjunctions and disjunctions are very different in this respect. A disjunction $p \vee q$ is downstream from p (q), while p (and q) should "already" be true before p & q can hope to be true. The word "consequence" has traditionally been used for both of these indiscriminately, but really we should distinguish consequences proper, like $p \vee q$ in relation to p, from what might be called "presequences,"[11] like p in relation to p & q.

The more we look into this distinction, the more presequences come to resemble parts. A table is partly blue just if it has a part that is wholly blue. Just so, a statement is partly true if it has a part that is wholly true. Other true implications do *not* confer partial truth on A. Consider *Goats eat cans and bottles*, on the assumption that they eat the first but not the second.[12] The conjunction is partly true because *Goats eat cans* is (wholly) true. The truth of *Goats eat cans or bottles* does not seem in this way to confer partial truth on *Goats eat bottles*.

Conferring partial truth is behavior characteristic of a part. Also characteristic of a part is the power to explain why the whole is *not* (fully) true. The house is not wholly blue because the door, which is part of the house, is red. A statement is not wholly true—it is false—if it is has a false part. Of course it's *sufficient* for falsity to have a false *consequence*. But that A has a false part *explains* why A is false. To explain the falsity of *Goats eat cans and bottles*, we point to the falsity of its presequence *Goats eat bottles*. To explain the falsity of *Goats eat bottles*, we do not appeal to the falsity of *Goats eat cans and bottles*. The falsity of $p \vee q$ does not shed much light on the falsity of p because it *consists* in part in p's falsity. We run into no such problem with p's falsity explaining that of p & q. For p's falsity certainly does not consist in part of that of p & q.

Conferring partial truth and explaining falsity are useful as diagnostics, but they don't tell us what it *is* for B to be part of A. The examples suggest that B, in addition to being implied by A, should not raise new issues, issues not already raised by A; B's subject matter ought to be *included* in that of A.[13]

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[10] Noted already by Ramsey in his review of the *Tractatus*.

[11] Pronounced similarly.

[12] The example of *Goats eat cans* is due Benj Hellie.

[13] Related definitions are given in Gemes (1997) and Fine (2013).

2 $B \leq A$ iff the inference $A \therefore B$ is

truth-preserving—B is true if A is
subject-matter preserving—B's subject matter is included in that of A

Next is to identify for each sentence S a subject matter **s** with some claim to be regarded as *its* subject matter, the one it is exactly about.

5 The subject matter of S

Our first clue here is that S's truth-value has got to supervene on how matters stand where its subject matter is concerned. It cannot change through changes in an aspect of reality that S is not even about. S's subject matter has got to be at *least* as fine-grained, it seems, as **whether or not S is true**. But it's not just *whether S* is true that supervenes on the state of things where its subject matter is concerned; *how* it is true supervenes as well. S cannot change in *how* it is true through a change in something it is not even about.[14] Pending a reason to go finer-grained than this,

3   The subject matter of S = **how S is true** or **ways of being true**[15]

Run through the last couple of definitions ((**1**) and (**3**)), (**2**) becomes

4 $B \leq A$ iff

A implies B, and
every way for B to be true is implied by a way for A to be true

Another way to put the same basic thought[16]:

5 $B \leq A$ iff

A implies B, and
worlds that differ in how B is true must also differ in how A is true

The number of stars includes **whether there are evenly many stars**, for instance, because you can't change the polarity of a number—its status as even or odd—without changing the number. **The number of stars and their total mass** includes **the number of stars**, because you can't change the number of stars without changing the number of stars and their total mass (Fig. 2).

This is all pretty abstract, so let's consider a bare bones model. The language is that of propositional calculus. A way for S to be true is a minimal model of S—a

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[14] If E is *The world will end in fire or ice* (Frost), then worlds where it ends in fire should differ where E's subject matter is concerned from worlds where it ends in ice.

[15] Also S's ways of being false, but we ignore this until the very end of the paper.

[16] The two are equivalent if subject matters are equivalence or similarity relations.

Fig. 2 Part-whole in propositional calculus

parts	non-parts
$p \wedge q \geq p$	$p \wedge q \not\geq p \vee q$
$p \wedge q \geq q$	$p \wedge q \not\geq p \leftrightarrow q$
$p \leftrightarrow q \geq p \rightarrow q$	$p \wedge q \not\geq p \rightarrow q$
$p \leftrightarrow q \geq q \rightarrow p$	$p \wedge q \not\geq q \rightarrow p$
$p \veebar q \geq p \vee q$	$p \wedge q \not\geq p \veebar q$
$pq \vee r \geq p \vee r$	$pq \vee pr \not\geq q \vee pr$
$pq \vee rs \geq q \vee s$	$pq \vee pr \not\geq pq \vee r$

partial valuation of the language all of whose classical extensions make S true, and such that smaller valuations have extensions making it false. So, $p \leftrightarrow q$ has as its minimal models (in a hopefully obvious notation), **p**, **q**, and $\overline{\mathbf{p}}$, $\overline{\mathbf{q}}$. (**2**) becomes, in this setting (where α and β range over minimal models of A and B),

6 $B \leq A$ iff

every α contains a β……………$A \therefore B$ preserves truth
every β is contained in an α……$A \therefore B$ is aboutness-preserving.

With the world role played by total valuations, and "having α in common" understood as having α as a shared subset, this becomes [compare (**5**)]

7 $B \leq A$ iff

every α contains a β
worlds with no β in common have no α in common

Implications that are content-parts are listed above, along with implications that are not.[17] So, for instance, q is part of p & q because {**q**} is a subset of {**p**, **q**}. $p \vee q$ is not part of p because the first has {**q**} as a minimal model, which is not included in any minimal model of p. $pq \vee rs$ includes $p \vee r$, because the latter's minimal models, {**p**} and {**r**}, are subsets respectively of {**p**, **q**} and {**r**, **s**}, both of them minimal models of $pq \vee rs$.[18]

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[17] $\veebar$ is exclusive disjunction.

[18] The definition as written makes $p \vee q$ part of p & q. This can be blocked by stipulating that B's minimal *counter*models $\overline{\beta}$, too, should be in each case included in some minimal countermodel $\overline{\alpha}$ of A. The definition then becomes: $B \leq A$ iff

(i) $\forall \alpha \exists \beta$ such that $\beta \subseteq \alpha$ (A implies B),
(ii) $\forall \beta \exists \alpha$ such that $\beta \subseteq \alpha$ (A's subject matter includes B's), and
(iii) $\forall \overline{\beta} \exists \overline{\alpha}$ such that $\overline{\beta} \subseteq \overline{\alpha}$ (A's subject anti-matter includes B's).

The reason $p \vee q$ is not part of p & q is that the former's only minimal countermodel assigns falsity to both p and q, whereas the latter's minimal countermodels assign falsity only to one or the other; an assignment to both is not included in an assignment to either.

6 Change

Back now to the main topic: what are some markers or signs of "parts as such," given that there is more to parthood than the merely structural requirements imposed by the standard axioms?

I do not claim to know all the reasons that Saturday strikes us as part of the weekend, but not of Friday. One striking difference, though, is the following. What happens on Saturday has ramifications for what happens on the weekend, but not for what happens on Friday. Change the course of events in Maine and you cannot help but change the course of events in New England. The Constitution changes when we tweak the 14th Amendment. A principle of *upward difference transmission* suggests itself:

8 y is part of x only if y cannot change (in specified respects) without x changing too (in those respects).

This is highly schematic, of course; one has to specify the relevant respects for each application. The sort of change involved might vary too; sometimes it is over time, sometimes between worlds, and there might be further possibilities.

Objects A bicycle frame y is part of a bicycle x, only if y cannot change intrinsically without x doing so as well. The frame cannot be bent or heated up while the bicycle sails on undisturbed.

Sets Here it is membership changes that percolate up. The set of flying things doesn't include the set of birds, for every new penguin changes the membership of the first with no effect on the second.[19]

Pluralities With pluralities, both sorts of variation—in intrinsic character, and membership—seem like they ought to percolate up. The Crown Jewels are among my possessions only if it reduces my possessions to destroy some of them, and rearranges my possessions to rearrange the Crown Jewels.

Properties When it comes to properties, we pass up their manner of possession.[20] Scalenehood includes the property of being a triangle because a figure cannot be identically triangular, in two worlds, but differently scalene between them. Negative charge includes charge, because a rod cannot lose charge while maintaining its negative charge. Grue is included in grue-and-slithery, because a snake that is grue here by being green and examined, there by being blue and not examined, has changed too in its way of being grue-and-slithery.

Changing the part results, in one category after another, in variation in the whole. With material objects, it is changes in intrinsic character that percolate up. With

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[19] Assume for these purposes that sets can survive changes of membership. Alternatively we could speak, perhaps, of one set being *replaced* by another.

[20] G is how F is possessed by a in w iff a is F in w by being G there.

sets, it is changes in membership. With properties, it is changes in manner of possession. A thing's way of being grue affects its way of being grue and slithery.

What should we expect to transmit up in the case of content-parts? Ways of being *true*, it must be, for a thing's way of being grue changes if and when *It is grue* changes in how it is true. *This is grue and slithery* includes *This is grue* for the same reason as *grue-and-slithery* includes *grue*. The difference is only that ways of being grue are replaced by ways for *This is grue* to be true. Our hypothesis is that

9 $A \geq B$ iff A implies B and

worlds that differ in how B is true must differ in how A is true.

This is the same as (**5**) above, derived this time from general considerations about parts, rather than the theory of subject matter.

7 History

Suppose we had taken the semantic route to content-parts first, and then got to wondering whether content-"parts" (defined in subject-matter terms) were properly so-called. That bicycles and the rest are not *about* anything is not encouraging in this respect. It makes content-parts look sui generis. To assuage these doubts we might attempt to reverse the order, starting with "regular" parts and asking whether content-parts carry anything over from them. That is what I have tried to do in this paper. Let me now go over the same ground more slowly, with the sui generis worry in mind.

An old observation about intrinsic properties[21] says that F is *not* intrinsic if parts can gain or lose F while the whole remains intrinsically the same.

10 F is intrinsic $\Rightarrow$ parts can't gain or lose it without intrinsic change in their containing wholes.

(**10**) of course has "intrinsic" on both sides. But suppose we juggle things around a bit. Isn't it also plausible that

11 y is part of $x \Rightarrow y$ can't change intrinsically without x doing so as well.

This has the advantage of putting the analysandum ("part") all on one side. (**11**) , which applies most immediately to material objects, now becomes the base camp for an assault on propositions. Lewis gets the ball rolling with his account of part/whole relations on sets. Pluralities, which have set-like aspects—the Fs have x as one of their number iff the set of Fs has x as one of its members—along with material-object-like features of the type appealed to in (**11**) are the obvious next step.

Properties, with all that has been written on ways of possessing them—intrinsically, at a time, by virtue of such and such now suggest themselves. Propositions are something like properties of worlds, which is how we got to (**9**). (**9**) is much like (**5**), which incorporates our attempted analysis of B's subject matter

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[21] Which I tried to work up into an analysis in Yablo (1999).

being included in that of *A* [by way of (**1**) and (**3**)]. The semantic route to content-parts, via subject matter inclusion, and the metaphysical route, via upward difference transmission, thus wind up in roughly the same place.

8 Objections and replies

1. WHAT IS PART OF WHAT? (Paul) You say Red is part of Scarlet. How do you know? The answer surely depends on our analysis of determinates and determinables. A determinable is the fusion or disjunction of its determinates, in my view. Scarlet on such a view is more plausibly part of Red than Red part of Scarlet.

REPLY: I agree that *if* Red is the disjunction of its shades—*if* it is Scarlet ∨ Crimson ∨ Ruby ∨ etc.—then Red is not part of Scarlet. This is one more case of disjunctions not being included in their disjuncts. Scarlet does not include Scarlet ∨ Crimson ∨ … for the same reason that goats eating cans does not include their eating cans or bottles or…. Our disagreement about whether Scarlet includes Red may be less about the relation of property inclusion, than about one of its relata: Red.

An initial worry about treating Red as disjunctive is that this insinuates every one of its shades into "what Red is." To come to an understanding of what Red is, one needn't know every shade in advance. (Some shades may be too specific for the likes of us *to* know.) Knowing the shades is required, though, if Red incorporates each of them separately.

History does not particularly smile on this hypothesis. Kant held that a subject-predicate judgment is analytic if the predicate is contained in the subject. Red had better not contain Scarlet, on this view, or *Red things are scarlet* comes out analytic, when it is not even true. Wittgenstein suggests that *P* entails *Q* only if *P* asserts inter alia that *Q*. Red had better not contain Scarlet on this view, either, or *This is scarlet* ceases to entail *This is red*. Logicians have wondered what is left, when a content-part (*Fa*, say) is subtracted from its containing whole (∀*x Fx*). No clear sense can be made of subtracting a set from one of its members, a fusion from one of the items fused, or a disjunction from one of its disjuncts.

This looks so far like an objection to treating Scarlet as part of Red. Alternatively the objection might be to treating it as a certain *kind* of part. To see what I mean by this, consider a tentative proposal of Lewis's about content-parts (Lewis 1988b). A proposition's parts can be identified with its implications, he initially conjectures. If propositions are sets of worlds, however, this can't work. Propositions' mereological relations were settled when we decided that subsets were parts of sets. Unfortunately they were settled in the wrong way from a propositional perspective; for smaller sets of worlds are "bigger" (stronger) qua propositions. The proposition expressed by *Tom is scarlet* is, to go by its subset relations, part of the proposition expressed by *Tom is red*. Which seems to Lewis to be getting things backwards. The idea of making Scarlet a part of Red gets the inclusion relations backwards too, even if it is forced on us by the identification of Red with a construct built on its shades.

I can imagine three responses to this sort of predicament. One is to follow the identification where it leads: if this is what the properties really are, then Scarlet is part of Red, strange as that initially seems. Lewis does a modus tollens; stronger propositions are *not* included in weaker ones, so the proposition that *P* cannot be the set of *P*-worlds. If we make it the set of worlds that *P excludes*, then the bigger set expresses the stronger proposition, and the set-theoretic and propositional notions of part are brought into harmony.

Neither option seems ideal—Paul's because of the bullet-biting involved, Lewis's for methodological reasons. Does Lewis see himself as *discovering* that propositions had been mis-defined? If so, he forgot the discovery quickly, for he returns, in subsequent papers, to the original definition, which is still the one that we use today.

A third option is not to choose. The *Tom-is-scarlet*-worlds are included qua *set* in the *Tom-is-Red*-worlds, but not qua *proposition*. Rather than making parthood a 5-place relation, it seems simpler to distinguish two notions of part, extensive and intensive. Paul could then have her cake and eat it too: Red extensively contains Scarlet, while being an intensive part of Scarlet.[22]

2. TRANSMISSION FAILURE 1: CARDINALITY (Sider, Kment)

An iron bar is infinitely long in both directions; it is green in one direction (to the left), unpainted in the other. Now we paint the section directly before us, thus extending the green part three feet to the right. A part changes intrinsically, by becoming green, yet the whole remains intrinsically the same. We would get the same result, intrinsically speaking, by shifting the bar three feet to the right.

Reply Suppose the affected section's material constitution is intrinsic to it—not only that it is made of iron, but that it is made of *that particular portion*, or quantity, of iron. Then it might be argued that the bar *does* change intrinsically; it was unpainted in its *m*-ish section, and now it is green there. Alternatively one could bypass the iron and make the section's own identity intrinsic to it, thus presumably making it intrinsic to the bar that it contains that section, suitably decked out. This was suggested, I think, by Ted Sider I won't pursue it any further except to note that Yablo (1999) allows identities to be intrinsic, though not perhaps to the extent needed.

If the bar does not change intrinsically when a single three-foot section is painted, then painting ten or a hundred sections can't change it either. And yet cardinality is not irrelevant. Suppose I paint *infinitely* many more additions to the green part,

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[22] I don't say that this simple two-way distinction is all we'd ever need, or that the distinction is clear, or that "intensive/extensive" is the right way to think of it. The richness of this terrain is brought out by Kratzer: "Paula didn't paint apples and bananas *apart* from painting a still life. Painting apples and painting bananas was part of her painting a still life, like my arms and legs are part of me" (Kratzer 1989, 609). The fruit-painting was part of painting the still life in what sense? The analogy with arms and legs suggests extensive parthood; the rest of the time she was painting a pheasant and wildflowers. Even if the still life consisted entirely of apples and bananas, though, the fruit-painting would still not an "extra" thing she does. Consider a simpler case: Paula paints a red tomato by painting a Scarlet Runner. Now its the thinner, less specific, event that plays the part role, which suggests we're talking about intensive part. See below for Kratzer's notion of lumping.

doubling my efforts each time to finish in an hour. When I'm done the bar is completely green. Compare what happens if I shift a duplicate bar infinitely many times to the right according to the same schedule. The bars are intrinsically indiscernible while the work is being done, so one might expect the same final result. But, while the first bar ends up completely green, its duplicate ends up simply *gone*; it has disappeared from the universe.

Now, as we know from Thomson's paradox of the lamp, these outcomes are not strictly entailed by what happens at earlier stages. It is no offense against logic if the moving bar winds up exactly where it began. We are moved rather by instinctive continuity assumptions. The original bar should maintain its position at $T = 1$ because it is stationary in the whole preceding interval [0, 1]. As for the second bar, a section that moved continuously would wind up infinitely far to the right; and there are no spatial positions infinitely far to the right of its initial position. Considerations of this kind seem to cast doubt on our (concessive) assumption that painting a three-foot section leaves the bar intrinsically unchanged. Indeed don't we now have a reductio?

(1) $\mathcal{B}$ did not change intrinsically between 0 and 1/2 (for contradiction)
(2) $\mathcal{B}$ did not change intrinsically between $1 - \frac{1}{2^k}$ and $1 - \frac{1}{2^{k+1}}$ $\forall k$ (analogy)
(3) Intrinsic unchanges never add up to an intrinsic change (transitivity)
(4) $\mathcal{B}$ never changes intrinsically before $T = 1$ [by (1)–(3)]
(5) $\mathcal{B}$ does change intrinsically at $T = 1$ (becoming all green)
(6) $\mathcal{B}$ does not change intrinsically at $T = 1$ [by (3) and (4)]
(7) Contradiction, so (1) is false; $\mathcal{B}$ did change intrinsically at the first stage.

The argument might seem to prove too much, for the *shifted* bar never changes intrinsically before $T = 1$ either, and yet it winds up completely gone; so it would seem also to follow, that even *moving* the bar changes it intrinsically. In response, line (5) seems wrong for the moving bar; a thing cannot be intrinsically changed at a time unless it exists then, and the bar presumably does not. Where would it be?

The step that does concern me is from (3) and (4) to (6). Transitivity of intrinsic sameness only gets us across *chains* of intrinsic duplicates. And there can be no chain leading from earlier stages to the bar at $T = 1$; the two are infinitely far apart. Do we really know that the sum of *infinitely* many unchanges is an unchange? Yes, cardinality seems at first irrelevant, but the Ross-Littlewood paradox argues the other way.[23] I am not sure the paradox has been mined yet for lessons about intrinsicness; that would be a good place to start if we want to get clear on Sider and Kment's interesting objection.

3. TRANSMISSION FAILURE 2: RELOCATION (Schaffer)

Here is how a part can change intrinsically without the whole doing so: you take it *out* of the whole first, leaving behind an exact duplicate. I can bend the handlebars all I want, if it's no longer part of the bicycle. Really the claim should be that parts cannot be modified without consequences for the whole *as long as they remain parts*

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[23] Allis and Koetsier (1991, 1995).

when the modification occurs. That has the notion of part on both sides, unfortunately. At best we could say that a relation R is parthood only if intrinsic changes in things (still) bearing R to x make for intrinsic changes in x.[24]

Reply[25] The relocation scenario has y changing non-intrinsically as well as intrinsically. This goes beyond what was contemplated. True, extrinsic changes were not forbidden either.[26] What exactly is y being asked to do, when it is asked to change intrinsically?

The simplest answer would be: only that. Y is not supposed to change in any way that is not implied by changing intrinsically. This can't be right, however, for "heating up" and "shrinking" are not implied by "changing intrinsically" either. Also implicitly permitted are the various *ways* of changing intrinsically, otherwise the permission counts for nothing. Imagine inviting someone to have some cake, and then objecting to their every attempt to do so on the basis that no mention was made of that piece in particular. Clearly if $\psi_1 \ldots \psi_n$ are the ways of φing, then one cannot consistently invite φing while forbidding each of the ψ_is.

You might worry that this opens the floodgates; for isn't φing-while-also-χing a way of φing, whatever χ may be? I certainly hope not, or one way of taking a piece of cake would be to take all of it; another would be to take your piece while trampling the rest into the ground.[27] Eating all the cake, or eating one piece while trampling the rest into the ground, cannot be defended as an exercise of the prerogative granted when you were invited to take some cake.

A rough test for whether ψ is a way of φing is the following: *I only $\varphi'd$* is compatible with *I also $\psi'd$*. That *I only took a piece of cake* is compatible with *I took the biggest piece* indicates that taking the biggest piece is acting under the authority granted by that earlier invitation. That it is *not* compatible with *I took one while trampling the rest* indicates that taking one piece and trampling the rest is *not* a way of taking a piece of cake.[28] Another rough test uses the by-locution. ψing is *not* a way of φing if the idea of φing by ψing is absurd, as the idea of eating cake by eating it with a hat on seems absurd.

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[24] This alternative has its own problems, Schaffer showed in email correspondence. But they are not the problems we're worried about now.

[25] I admit it took me a while to appreciate the force of this worry. My first thought was that one just had to be more careful about the time: y is part of x over a certain interval of time only if intrinsic changes in y over that interval are passed up to x. No such luck, however.

[26] Nor could they have been forbidden, without the whole of nature outside of y having to stop in its tracks.

[27] Lewis (2000).

[28] See the "Dialogue with a Lunatic" in Kratzer (1989). "Lunatic: What did you do yesterday evening? Paula: The only thing I did yesterday evening was paint this still life…Lunatic: This is not true. You also painted these apples and you also painted these bananas. Hence painting this still life was not the only thing you did yesterday evening." ψ is not an "extra" thing done, for Kratzer, if φ "lumps" ψ, that is to say, every actual φ-supporting situation is a situation in which ψ. holds. A disjunction, for instance, lumps its true disjunct. (To avoid overdetermination worries, as when a disjunction has two true disjuncts, we should say that φ lumps ψ if there *exists* a φ-supporting situation each of whose φ-supporting sub-situations also supports ψ.)

Now let φ be changing intrinsically. The ways of changing intrinsically are, to a first approximation, the ψs such that *y only changed intrinsically* is compatible with $y\ \psi'd$. Changing shape is a way of changing intrinsically, because to change shape does not mean you did something else besides change intrinsically; a thing changes intrinsically *by* changing shape. Moving outside of x and *then* changing shape is not by these criteria a way of changing shape. A better way to put the proposal, then, would be this:[29]

12 y is part of x at time T only if

> any way for y to change intrinsically at T requires x to change intrinsically as well; there is no way for y to change intrinsically that allows x to remain intrinsically as it was.

It may seem strange to rest so much on a notion like *way* of changing, which seems rather more esoteric than that of material part. But again, I am not attempting an analysis. The appeal to ways brings out a further unity, moreover, in our mereological notions. For ways figure as well in the definition(s) of content-part, for instance this, slightly reformulated from above:

13 $B \leq A$ iff

> worlds verifying A must verify B (A implies B)
> worlds verifying A the same way must verify B the same way

A quick reminder of how this works. *Goats eat cans or fly planes* is implied by, but not part of, *Goats eat cans*. Why is it not a part? Imagine worlds w and w^* in both of which goats eat cans and fly planes. Their can-eating predilections are the same in the two worlds. In w, though, they fly jokey prop planes in a circus act, as opposed to supersonic jets in w^*. The worlds differ in how it is true that goats fly planes or eat cans, but not in how it is true that they eat cans. *Goats eat cans* is part of *Goats eat cans and fly planes*, because if goats' gustatory relation to cans changes, between two worlds where goats fly planes, the way goats eat cans and fly planes has got to change too.[30]

4. Transmission failure 3: Determinables (Skow, Dorr)

You say that B is part of A only if worlds differing in how they verify B must differ in how they verify A. Worlds can differ in how Tom is colored, though, without differing in how Tom is red, simply because Tom is green in those worlds.

Reply: You're right. Better would be: worlds differing in how Tom is colored must also differ in how Tom is red, *if Tom is indeed red in them*. Equivalently, worlds differing in how they verify B cannot agree in how they verify A.

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[29] Thanks to Louis DeRosset for urging this line of response.

[30] Conjecture: *Part* and *way* are duals in the case of content parts. An initial hypothesis along these lines: C is a way for A to be true iff $\neg C \leq \neg A$, and C is part of all stronger Ds such that $\neg D \leq \neg A$.

14 $A \geq B$ iff A implies B and

worlds that differ in how B is true cannot agree in how A is true.

5. OVERGENERATION (Yablo)

Now you have set the bar too low. t.[31] I take it that *Tom is red or unripe* is *not* supposed to be part of *Tom is scarlet*. If it were, then *Tom is scarlet* would be partly true in a world where Tom was unripe and therefore green. It fulfills nevertheless the revised condition (**14**): worlds differing in how *Tom is red or unripe* is true cannot agree in how *Tom is scarlet* is true. The reason is this. To agree in how *Tom is scarlet* is true, they will have to be worlds in which Tom is not unripe. To differ in how Tom is red or unripe, they will have to differ in how Tom is red (since Tom is not unripe). To differ in how Tom is red, they will have to differ in how Tom is scarlet, contrary to our initial stipulation.

Reply A test is needed that favors determinables (*Tom is red*) of *Tom is scarlet* over disjoint disjunctions like *Tom is red or unripe*. The test is not going to be in terms of ways of being true, because *Tom is red* has the same truth-grounds as *Tom is red or unripe* in worlds where Tom is scarlet. Do they have different falsity-grounds, perhaps, in worlds where Tom is not scarlet? Yes they do, for *Tom is red or unripe* could be false by way of Tom being yellow and ripe. That is not a way for *Tom is scarlet* or *Tom is red* to be false, because the ripeness is beside the point. Imagine worlds where Tom is barely ripe and overripe, respectively, while maintaining its shade of yellow. These are going to differ in how *Tom is red or unripe* is false without differing in how it is false that Tom is scarlet. The proposal this suggests is

15 $A \geq B$ iff A implies B and

(i)   worlds that differ in how B is true cannot agree in how A is true
(ii)   worlds that differ in how B is false cannot agree in how A is false.

This marks a further departure from material parthood, since there is nothing in (**14**) corresponding to clause (ii) in (**15**). Should there be? I am going to have to leave these questions for now, not sure we have found a feature common to all varieties of part/whole, but encouraged enough to keep trying.

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[31] Assume for these purposes that an unripe tomato is green.

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