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IFS, ANDS, AND BUTS: AN INCREMENTAL TRUTHMAKER SEMANTICS FOR INDICATIVE CONDITIONALS

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I. Ifs

Indicative conditionals $A \rightarrow C$ are epistemic and in some sense subjective, according to current orthodoxy. Their assertability (or assentability) in a context is thought to reflect the properties of credal or information states, far as these might be from the apparent subject matters of *A* and *C*. Indicative conditionals do not even *say* anything, if the triviality arguments are to believed, of a type to be regarded as true or false. The most they can aspire to is to *seem* right to someone with the right kind of epistemic perspective. This is the view of, among many others, Ernest Adams, Allan Gibbard, Jonathan Bennett, and Dorothy Edgington.

But of course, information states are not pulled out of a hat. They derive in many cases from empirical assumptions that are evaluable as true or false.[1] If those assumptions are mistaken, indicative conditionals based on them are liable to be mistaken too, however acceptable at the time of utterance. Even if conditionals in the first instance express states of mind, they nevertheless in the second instance track the external facts to which those states of mind are answerable.

Subjectivists wrestle with this in various places. Adams discusses the case of Alice collecting some possibly poisonous mushrooms. *You will die if you eat those,* Boris tells her, in the belief that they are poisonous, and she refrains. Later, when the mushrooms turn out to be harmless, *You would have died if you'd eaten those* strikes us as wrong. No surprise there, you may say; counterfactuals track actual underlying mechanisms, whether anyone knows about them or not. What *is* slightly surprising is that the original indicative now seems wrong too. As does Boris's subsequent claim (he hasn't heard they were safe) that Alice did die, if she did eat the mushrooms.[2] Boris's conditional probabilities, based as they were on bad information, are relevant to

[1] "We can [oftentimes] identify evidential relevance [of the antececedent to the consequent] with beliefs about connections between the facts" (Stalnaker 1984).

[2] "The man in the street usually says that ... the [indicative] statement is plainly false because the mushrooms were actually non-poisonous" (Adams 2005, 7).

acceptability, but not, it seems, to truth. Adams is aware of these judgments but it is not clear how he wants to account for them.

An opposite sort of case, due to Morgenbesser, features an initially *un*acceptable conditional that is vindicated by events. Alice is about to toss a coin; Boris was going to put his money on heads, but elects in the end not to bet. He would have won, Cloris rightly remarks when the coin comes up heads. This conditional is of course subjunctive. But speculation before the toss that he *will* win if he bets on heads is vindicated too, as is a later past-tense indicative to the effect that Boris *did* win if he, despite appearances, *did* bet on heads. Or imagine, as Edgington does, that Alice is taking a cab to the airport. The driver has a premonition, which he keeps to himself, that she will die if she gets on that plane. Alice arrives too late to be boarded and the plane goes down without her. The cabbie admittedly did not *know* that things would play out as he imagined. He was surely right, though, when he (however irresponsibly) said to himself that Alice will die if she gets on the plane.[3] He is also right if he judges later, not having heard of the crash, that Alice did die if she did get on the plane.

Not all conditionals are as factual-seeming as these ones. But wholly epistemic conditionals are not so common, either. Stalnaker suggests the following: a reliable oracle tells you that all the sentences on a certain whiteboard agree in truth-value. As it happens, the first sentence is *Goats eat cans*, and the fifth is *Fish never sleep*. You conclude that if goats eat cans, then fish never sleep. The plausibility of this conditional is entirely information-based. Between the states of affairs themselves, there are no useful relations whatever. There are cases and cases, then. Epistemic and factual are not sui generis categories, one assumes, but variations on a theme. What is the theme?[4]

II. Subjective/Objective

Conditionals are evaluated in light of bodies of information, according to subjectivists—that the coin came up heads, that the mushroom was poisonous, that the sentences agree in truth-value. They look to the properties of this information for the explanation of relative objectivity.

[3] "Someone who did make the corresponding forward-looking indicative conditional judgements in advance, however unreasonably, could claim to have been right, when the coin did land heads, and the plane did crash" (Edgington 2013).

[4] Precis of the next section: Information-based conditionals become more factual according to the qualities of the information. This is the usual way to bring the two types of conditional together. It is not the only way, however, nor is it clearly the best.

Is the information true, or only believed to be? Does it stand out in itself or does it merely reflect our epistemic position? Does it *explain* why $A\&\neg C$ must fail, as the coin coming up heads explains why Boris cannot both bet on heads and lose? Or is it only *evidence* that the combination doesn't obtain, as *Goats eat cans* and *Fish never sleep* appearing together on that whiteboard suggests, without shedding any light on the matter, that *Goats eat cans* and *Fish sometimes sleep* are not true together. These are some of the issues Stalnaker seems to be raising when he says,

> If we could filter out those aspects of our epistemic situation which derive more from our parochial perspective and less from the way we take the world to be, we might be able to explain the acceptance of conditional *propositions* in terms of [the indicatives] that would be acceptable in ideal contexts which abstract away from these aspects (Stalnaker 1984).

Stalnaker speaks of "filtering out aspects of our epistemic situation deriving from our parochial perspective." This idea has a history in philosophy. It calls to mind Bernard Williams's discussion in Williams (2005) of the *absolute conception of reality*. To conceive the world absolutely, Williams says, is to conceive it in a way that does not cater to any particular sensibility or point of view. There is no room for sweetness in the absolute conception, because (i) sweetness reveals itself only to creatures with a certain sensory apparatus, and (ii) it owes as much to the state of that apparatus as to the external world.[5] Geometrical properties do better in these respects. Cylindrical pillars may seem to bulge slightly, and railway tracks may seem to converge in the distance, but we know how to control for these factors to arrive at a shared assessment of objects' actual spatial features.

Are conditional judgments like judgments of taste, or of shape? One can imagine arguments on both sides. Favoring the first hypothesis, perhaps, is that (i) if-appearances present themselves only to creatures with conditional credences, and (ii) they reflect those credences as much as goings-on in the outside world. Favoring the second is that conditional credences, like shape-appearances, are educable and apt to become less idiosyncratic as they become better informed. An up or down answer to this sort of question is not to be expected. Some conditionals are relatively objective; some are more parochial. The most we can hope for is to get them under the same theoretical umbrella.

There are two obvious things to try if the goal is to make X and Y look like variations on the same theme: start with X and try to make it more Y-like, or vice versa. The options in this case are to start with subjective/epistemic conditionals and make them less parochial, or to start with absolute conditionals and try to work some perspective into

[5] Williams (2005):49–50.

them. The first strategy has the advantage that we have some idea how to do it. Epistemic conditionals are everywhere, and one sees more or less how "beliefs about connections between the facts" can come to play a larger role in the information states supporting them. One has by contrast little idea of what an absolute conditional is supposed to look like. I want to try nevertheless the strategy of reaching first for the absolute end of the stick. The next section tries to explain why.[6]

III. Absoluteness

To rise above our parochial perspective, we might try *embellishing* that perspective so that proprietary information gets lost in the crowd. $A \rightarrow C$ will be true, on this approach, just if it is acceptable "to a perfectly rational thinker with unlimited mental powers and knowledge of all the facts."[7] $A \rightarrow C$ will be acceptable to this thinker if and only if he

> would come to accept the proposition expressed by *C* if he were to add the (hypothetical) belief that *A* to a non-question-begging but otherwise unrestricted stock of true beliefs and make whatever minimal adjustments are necessary for the sake of consistency (without sacrificing the belief that *A* in the process) (186)

What counts as a "non-question-begging but otherwise unrestricted stock of true beliefs"? Our thinker will be guided by "general beliefs...central to our conception of the world" and "beliefs concerning 'relevant conditions'," but not "beliefs concerning the truth-values of *A* and *C*, or other beliefs which depend on them or on which they depend," except when one believes *A* to be true and *C* false (203). This is more of a wish list, than a plan. How are these judgments to be made (e.g. of "centrality" and "relevant conditions") if not on the basis of restricted world knowledge? The strategy that embellishes our epistemic situation—that gives us knowledge of "all the facts," except facts too caught up with whether *A* and whether *C*—cannot really be carried out, because "too caught up with" has no clear non-epistemic meaning.

[6] Precis of the next section: If conditionals are information-based, what kind of information $\mathcal{I}$ is an *absolute* conditional based on? $\mathcal{I}$ should be neutral, other things equal, on whether *A*, and also on whether *C*. That still leaves lots of options, obviously. Would it help for $\mathcal{I}$ to be *maximal* among information-states neutral on *A* and *C*? Not much; there are are lots of option here too. There is no such thing as the maximal subtheory *T* of $A\&X\&C$ that is neutral on *A* and *C*. Shall it contain $X \equiv C$ or $X \vee C$? One or the other, by maximality, but not both, for the two together imply *C*. So we are left again with a choice. The only real option, it seems, is for absolute conditionals to be based on NO information.

[7] Pendlebury (1989):185.

The only other strategy I can think of is to *impoverish* our epistemic perspective by allowing ourselves no factual information whatever. This is one of the attractions of the material conditional reading, I take it. One can evaluate $A \supset C$ just on the basis of A and C; nothing need be known about how A connects up with C. Which is of course also the problem with the material conditional reading. As already mentioned, we would like if possible to *bracket* the truth-values of A and C, concerning ourselves rather with the relation between them—how A connects up with C. This makes little sense if $A \rightarrow C$ is truth-functional. Why would we want to bracket the truth-values of A and C if they are the sole determinants of its truth-value?

Now we seem to have painted ourselves into a corner. A's relation to C is an empirical matter, and empirical matters were to be avoided by making $A \rightarrow C$ truth-functional. This line of thought rests on confusion, however. How *we* determine $A \rightarrow C$'s truth-value is one thing; how the world determines it is another. Absoluteness is threatened only if the *second* is not objective. The problem with $A \supset C$ is not that the world doesn't decide its truth-value, but that it decides it the wrong way, focussing on A and C rather than the relations between them. Do the relations obtain? That is of course an empirical matter. Our concern is with what the relations *are* on which the truth of $A \rightarrow C$ depends. It's the *identity* of those relations that has to be kept non-empirical.

Truth-functionality is certainly one way to arrange that $A \rightarrow C$'s demands on a world can be identified non-empirically. But it is not the only way. It would be enough for absoluteness if $A \rightarrow C$ were truth-*condition*-functional, that is, if what $A \rightarrow C$ asked of a world were a function of what A and C asked of it. We know in fact of conditionals like this. If p and q are thoroughly independent, then what $p \rightarrow (p \& q)$ asks of a world is that q should hold in it. I myself am inclined to read *If you're tired, that makes two of us* so that its truth-conditions are precisely that I am tired. There may be other ways of understanding it as well; your tiredness could function as *evidence* that I am tired. But that is OK with the absolutist, indeed, it is to be expected. Absolute conditionals are meant to be a specially perspicuous *subclass* of indicative conditionals. Other conditionals are obtainable, we hope, by relaxing the conditions defining that subclass.[8]

IV. Methodology

How will these conditions be found? Absolute indicatives are defined so far by a goal, not a set of truth-conditions. Is that a problem? I see it more as an opportunity. We are trying to work out how absolute indicatives, if there are going to be any, will have to behave. $A \rightarrow C$ is not being

[8] Precis of the next section: a method is sketched for identifying the conditions that $A \rightarrow C$ will have to satisfy, if it wants to be absolute.

read absolutely, if it asks different things of a world depending on factors that ought not to matter. Suppose a parameter X can be identified that reflects something parochial: speaker information, say, or speaker preferences, or how the speaker "groks" the conditional. And suppose that $A \rightarrow C$ appears to ask different things of a world, depending on the value X takes. Then, since that kind of dependence is not allowed when $A \rightarrow C$ is read absolutely, the sought after reading will have to either ignore X or freeze it at some canonical value.

An example might be this. Speakers have a certain amount of discretion about how much weight to lay on (i) C's stand-alone plausibility, as against (ii) the comparative plausibility of C given A and on $\neg A$. Sometimes A is expected to bear more favorably on C than $\neg A$ does. Other times it's allowed that C holds *despite* A or *regardless* of whether A. (This may or may not be marked linguistically, as in *C even if A* or *C whether or not A*.) Conditionals read the second way—so-called non-interference conditionals—owe their plausibility almost entirely to C. All we learn about A is that it doesn't pull the rug out from under C, and sometimes not even that. There are clearly a range of possibilities here, according to how A's bearing on C is traded off against C's intrinsic plausibility.

Suppose that Donald Trump is reliably wrong on foreign policy matters. He may say, for instance, that any idiot could have negotiated a better deal with Iran than Obama did, when in truth (let us suppose) Obama got the best possible deal. Consider this conditional: *Obama got the best deal, if Trump says so.* I feel myself pulled two ways when I try to evaluate it. Sometimes I want to read it as a non-interference conditional; it is true simply because Obama got the best deal, whatever Trump may say. A reading that takes the antecedent more seriously will have it turning in part on whether Trump speaks the truth on matters like the Iran deal. Then it is apt to strike me as false; Trump is so anti-reliable on these matters that Obama probably did *not* get the best deal, if Trump thinks he did.

This is the kind of interpretive discretion that will have to be blocked on an absolute reading. One approach would be to lean exceedingly far in the direction of non-interference conditionals. By this I mean that we could ignore A entirely, and treat $A \rightarrow C$ as just a long-winded way of saying that C. If our one and only goal were to avoid awkward tradeoffs between A's relevance to C and C's inherent plausibility, confining our attention to C would accomplish that.

Imagine though that we wanted the option of taking A seriously, as surely we do. Then the issue becomes how seriously to take it. Again we turn to absoluteness for guidance. The only non-discretionary response to *How important is A's bearing on C, as against C itself?* is this: we should *ignore* C's antecedent plausibility, and let the issue turn entirely on the link (or not) between *A* and *C*—in this case on whether Trump is right or wrong about Iran. Assuming again that he is wrong, the conditional strikes us as false. Consider next *If Trump says that Obama got the best deal, he is right*. This time the missing link is that Obama got the best deal, and the conditional seems much more plausible.[9]

Plan of the paper: The problem of absolute "if" is reviewed in the next section. Absolute conditionals will have to be *incremental*, I claim. Their truth-value turns on that of *C*'s *surplus content* with respect to *A*. Section 6 pits incremental conditionals against Lewis's Triviality Proof. Section 7 looks, through the lens of the Ramsey Test, at the state of mind underlying acceptance of indicative conditionals generally (it can be skipped without loss of continuity). Section 8 reframes the Triviality question in terms of Kaufmann's distinction between global and local styles of interpretation. Section 9 considers the idea of surplus content in its own right. A connective $\sim$ is introduced—something like the inverse of conjunction—such that $C \sim A$ is the remainder when *A* is subtracted from *C*. Affinities between remainders and conditionals are noted in Section 10. Formal details are supplied in Section 11 and an Appendix. Section 12 introduces a notion of "conservativeness" borrowed from generalized quantifier theory. Absolute conditionals are not conservative, and, it is argued, marketplace conditionals are not conservative either, as existing theories say they should be. The remaining sections explore the prospects for understanding marketplace conditionals on the model of incremental conditionals.[10]

V. Surplus Content

The notion of surplus content originates in 20th-century philosophy of science. Goodman wanted to distinguish inductive confirmation—the way *This raven is black* confirms *All ravens are black*—from "mere content-cutting"—the way *The coin came up heads* confirms *It always comes up heads*. The difference is supposed to be that *This raven is black* confirms

[9] It might be objected that *C*'s truth-value *is* sometimes dispositive, notably in *A*-worlds. How on the proposed policy is it to be explained why $A\&C$ entails $A \rightarrow C$, and $A\&\neg C$ entails $\neg(A \rightarrow C)$? This assumes that the only thing distinguishing $A\&C$-worlds from $A\&\neg C$-worlds is whether *C*. Another thing distinguishing them is whether "the link" obtains, in this case whether Trump is right or wrong about Iran. If the link *L* between *A* and *C* is such that *L* holds in *A*-worlds where *C* holds, and *L* fails in *A*-worlds where *C* fails, then *C*'s antecedent plausibility is safely ignored. Such an *L* will be introduced below.

[10] Precis of the next section: Two notions of surplus content are distinguished, associated, respectively, with Popper and Goodman. Popper's notion, $A \supset C$, takes the shortest path from *A* to *C*, if longer means *stronger*. Goodman's pick takes the shortest path if longer means *more circuitous*. Goodman's approach is more plausible but the notion of shortest-qua-directest (straightest?) logical path is so far undefined. An initial suggestion is made as to how straightest logical paths might be understood.

the surplus content of *All ravens are black*, viz. *All other ravens are black*, while *The coin came up heads* does not confirm *It comes up heads on every other toss* = the surplus content of *It always comes up heads*.

Popper and Miller use the surplus content idea *against* Goodman in a 1983 letter to *Nature*. If inductive evidence has got to support surplus content, then there is no such thing, they maintain. Their argument has two premises. First, *H*'s surplus content relative to *E* is $E \supset H$.[11] Second, evidential support is to be understood as positive probabilistic relevance. Far from supporting $E \supset H$, *E* bears *negatively* on it, since $E \supset H$ is implied by $\neg E$.

If the absolute conditional is to be understood in terms of surplus content, and the surplus content of *C* over *A* is $A \supset C$, then $A \rightarrow C$ read abolutely is just $A \supset C$. Is this a plausible result? What do we think of $\supset$ as a candidate for the role of absolute "if"? A point in its favor is that $A \supset C$ takes the shortest path from *A* to *C*, in this sense: it is the weakest statement that can be plugged in for *R* in the enthymeme $A, R \therefore C$ to obtain a valid argument. But, as already noted, $A \rightarrow C$ should concern itself with the *relations* between *A* and *C*, rather than the two taken separately. It should not be true just because *A* is false, or because *C* is true.[12]

Now, it is not as though $A \supset C$ can *only* hold because *A* is false or *C* is true; *Goats eat cans* ⊃ *They eat cans and tinfoil* might surely be true because goats eat tinfoil. Goats eating tinfoil is a third sort of truthmaker for the material conditional, over and above their not eating cans and their eating cans and tinfoil. The problem with $A \supset C$ is not that it *can't* hold for the *right* reasons, but that it *can* hold for the *wrong* reasons. $A \supset C$ will in fact be playing a crucial role. Absolute $A \rightarrow C$'s reasons for being true will have to imply truthmakers for $A \supset C$, or we lose modus ponens. Its reasons for being true may indeed *be* truthmakers for $A \supset C$. It is just that they'll have to be a certain *kind* of truthmaker, the kind that targets the space *between* *A* and *C*, rather than either taken individually.

An analogy may be helpful. Suppose we were looking, not for *C*'s added value relative to *A*, but 33's added value relative to 8. This added value is 33–8 = 25. Now, 25 has no more to do with 33, or with 8, than with any other numbers. That the *numerals* 33 and 8 figure in a *characterization* of 25 does not mean that the numbers 33 and 8 figure in the number 25. What goes for 33 and 8 goes for *C* and *A* as well. *C* and *A* do of course figure in our *characterization of C*'s surplus content over *A*. But the contents of those sentences do not carry over into in the surplus content of one over the other, any more than 33 and 8 are lurking unnoticed in 25.

[11] Hempel seems to be have this conception of surplus content in mind when he says that $E \supset H$ "has no content in common with *E*," since its disjunction with *E* is a logical truth (Hempel 1960, 465).

[12] "Just because *A* is false" is a nod in the direction of truthmakers. Truthmakers for $A \rightarrow C$ should ideally be neutral on whether *A*.

How can the shortest path from A to C not be $A \supset C$, if every other enthymeme-completer is stronger? *Stronger* does not mean *longer*. A longer path from A to C is one that is more *indirect* or *circuitous*. The path from p to $p\&q$ than which every other is stronger is $\neg p \vee (p\&q)$, The path from p to $p\&q$ than which every other is longer is q. Likewise the shortest path from a generalization's *observed* instances to the generalization as a whole is by way of its *other* instances. This is presumably what Goodman had in mind, since he says that a black raven Rudy reflects favorably, not on *All ravens are black if Rudy is black*, but *All ravens other than Rudy are black*.

What would a shortest path conditional look like in general? This is what we are coming to, but we can at least say the following. It should be true for reasons that block $A\&\neg C$ "as such." Anticipating a bit, let a targeted truth maker for $A \supset C$ be a fact combining with A to imply C that's consistent with A and that "makes the most of A," getting all the help from A that it can. $A \rightarrow C$ is true (read absolutely) where a targeted truthmaker obtains for $A \supset C$, and because of that targeted truthmaker. (It is false where and because a targeted truthmaker obtains for $A \supset \neg C$.) Absolute $A \rightarrow C$ expresses the (possibly partial) proposition whose truth goes with the disjunction of targeted truthmakers for $A \supset C$, and whose falsity goes with the disjunction of targeted truthmakers for $A \supset \neg C$.[13]

To explain this properly, especially the bit about making the most of A, is not trivial (see Section 11). But it is easier than explaining what perfect thinkers are and what constitutes a "minimal A-preserving adjustment to a non-question-begging but otherwise unrestricted stock of true beliefs." (There is even a literature on the topic, growing out of Wittgenstein's question about what is left over when arm-rising is subtracted from arm-raising.[14]) The idea will be that R takes the shortest/straightest path from A to C just if R is the enthymeme-completer that takes maximal advantage of A.[15] It is this shortest-path R that an un-parochial observer looking for $A \rightarrow C$'s factual basis will be drawn to.[16]

[13] If a world contains targeted truthmakers both for $A \supset C$ and $A \supset \neg C$, then depending on the application we may want to regard $A \rightarrow C$ either as true and false, or neither true nor false.

[14] Hudson (1975), Jaeger (1976), Humberstone (1981), Fuhrmann (1996, 1999), Humberstone (2000).

[15] Yablo (2014)

[16] Precis of the next section: Incremental conditionals have truth-conditions. Indicative conditionals supposedly do not, due to certain well-known triviality results. These results assume ADAMS' THESIS—$\pi(A \rightarrow C) = \pi(C|A)$—which implies INDEPENDENCE: $A \rightarrow C$ is probabilistically independent of A. INDEPENDENCE is prima facie puzzling; why can't an oracle tell us that A and $A \rightarrow C$ stand or fall together? When it comes to incremental conditionals, INDEPENDENCE is downright absurd. $A \rightarrow A\&X$ on an incremental interpretation is generally just X; for A to be independent of $A \rightarrow A\&X$ thus means that A is independent of X. But X could be almost anything! INDEPENDENCE implies, absurdly, that everything is probabilistically independent of virtually everything else.

VI. Triviality

Remainders are propositional and fact-stating. Indicative conditionals are supposedly neither, due to the triviality results. Let's review the first and simplest of these, due to Lewis. $A \rightarrow C$'s propositional probability would have to be governed by standard probabilistic laws. $\pi(A \rightarrow C)$ should by the Law of Total Probability be

(1) $\pi(A \rightarrow C|B) \times \pi(B) + \pi(A \rightarrow C|\neg B) \times \pi(\neg B)$.[17]

$\pi(A \rightarrow C|X)$, we assume following Adams, is $\pi(C|A\&X)$.[18] So $\pi(A \rightarrow C)$ can also be written as follows:

(2) $\pi(C|A\&B) \times \pi(B) + \pi(C|A\&\neg B) \times \pi(\neg B)$

The problem is that this yields different results, some of them clearly unwanted, for different values of *B*. Lewis, setting *B* equal to *C*, gets that $\pi(A \rightarrow C)$ is

(3) $\pi(C|A\&C) \times \pi(C) + \pi(C|A\&\neg C) \times \pi(\neg C)$

which, since $\pi(C|A\&C) = 1$ and $\pi(C|A\&\neg C) = 0$, simplifies to $1 \times \pi(C) + 0 \times \pi(\neg C) = \pi(C)$. $A \rightarrow C$ is equal in probability to *C*? How could that be? What is the antecedent doing there, if it is just going to be tossed out?[19] Other choices of *B* lead to equally strange results. If *B* is $A\&C$, we get that $\pi(A \rightarrow C) = \pi(A\&C)$. If *B* is $A \supset C$, we get that $\pi(A \rightarrow C) = \pi(A \supset C)$. If *B* is $A \vee C$, $\pi(A \rightarrow C)$ becomes $\pi(C|A) \times \pi(A \vee C)$ + undefined. If *B* is $C \equiv A$, we get $\pi(C \equiv A)$ + undefined.[20]

This is not as big a problem as it once seemed. Opinion has been converging anyway on a certain response to Lewis's argument, one that seems particularly compelling from an incrementalist perspective. The response starts with a number of seeming

[17] Assuming $\pi(B)$ is not 0 or 1.

[18] Adams (1975)

[19] We'll be charging current theories with a related, though lesser, offense, viz. tossing out the information encoded in *C*'s behavior *away* from the antecedent.

[20] One natural thought at this point is that truth-functional combinations of *A* and *C* are bad things to partition on when assigning a probability to $A \rightarrow C$, for they in different ways screen off or otherwise distort *A*'s influence on *C*; more on this below.

counterexamples to $\pi(A \to C) = \pi(C|A)$ (the THESIS). Here is a stripped-down version of Vann McGee's game-show case.[21]

You are a contestant on "To Tell the Truth." Your job is to identify the panelists, all in disguise, on the basis of how they respond to cleverly posed questions. Convinced as you are that #1 is Holmes, how likely is it that if #1 says the killer was Mrs Hudson, the killer was Mrs Hudson? *Highly* likely, you might think, since Holmes is hardly ever mistaken. For that very reason, #1 is not expected to accuse Mrs. Hudson in the first place. (Mrs Hudson is the kindly landlady.) Indeed if he *were* to accuse her, rather than accept that she was guilty, we would reconsider our view of #1 as Holmes. The probability of *If #1 says the killer was Mrs Hudson, the killer was Mrs Hudson* is thus lower on the assumption of its antecedent than considered alone. The THESIS can't allow this, for it entails the principle of[22]

INDEPENDENCE $\pi(A \to C|A) = \pi(A \to C)$

INDEPENDENCE seems clearly wrong in the game show case; *'Mrs Hudson'* → *Mrs Hudson* is less probable, we have said, conditional on its antecedent, than independently.[23]

Now, whatever one thinks of this particular example (there will be others below), the idea of $A \to C$ having to be probabilistically independent of A ought to strike us as surprising. Is there precedent for this sort of thing elsewhere in logic/epistemology, where sentences X and Y are prevented *just by their form* from bearing evidentially on each another? I know only of precedent for the opposite. A is favorably relevant to $A\&C$ just because it recurs as a conjunct. A is relevant to $A \vee C$ because it recurs as a disjunct. One might perhaps expect A to be relevant to $A \to C$, since A recurs as an antecedent. This doesn't follow, let's agree. But look what we are being asked to accept: that the *opposite* follows. What could possibly prevent an oracle from informing us that $A \to C$ is (im)probable given A? (Maybe A and $\sim(A \to C)$ are both on the whiteboard that we've been told contains statements all of the same truth-value.)

If INDEPENDENCE holds only sometimes, that points in the direction of incrementalism, for the *nature* of the mistake comes out most clearly on an incrementalist perspective. $A \to C$ read incrementally is just a completely new proposition, whose probabilistic relations with A are as may be. If C is $A\&Z$, for instance, then $A \to C$ is $A \to (A\&Z)$, which is equivalent for present purposes to Z. INDEPENDENCE says that A and Z are probabilistically independent. But A and Z could be anything; they were chosen

21 Adams, Stalnaker, and Pollock had cases like these, but may not have attached the same importance to them. An example of Kaufmann's is given later.

22 Proof of INDEPENDENCE from the THESIS: $\pi(A \to C) = \pi(C|A) = \pi(C|A\&A) = \pi(A \to C|A)$.

23 This is an indicative analogue of the so-called conditional fallacy, discussed in the next section.

more or less at random. If nothing was ever favorably relevant to anything, $A \to C$'s failure to express a proposition would be the least of our problems.[24,25]

VII. Independence and Acceptance

Consider what is involved in *accepting* a conditional, absolute or otherwise. The usual story, or story-outline, is due to Ramsey. "If two people are arguing 'If *A* will *C*?' and both are in doubt as to *A*, they are adding *A* hypothetically to their stock of knowledge and arguing on that basis about *C* ... We can say that they are fixing their degrees of belief in *C* given *A*" (Ramsey 1931, p. 249). Ramsey attempts to indicate here the state or condition of mind that goes with accepting a conditional more or less strongly. What *is* that state of mind, in his view? The quote doesn't pin it down exactly, but the main candidates appear to be

(DIS) the strength of one's disposition to believe *C*, on learning *A*.
(CFL) the confidence one would have in *C*, on learning *A*.
(CCR) one's conditional credence in *C* given *A* (that is, $\pi(C|A)$).

The last of these, (CCR), is *Adams' Thesis*, or just[26]

THE THESIS $\pi(A \to C) = \pi(C|A)$

The first is dispositionalism, sometimes advocated by Stalnaker:

[24] Another proof of triviality, due to Richard Bradley, proceeds from a weaker assumption, *Preservation*: If $\pi(A) > 0$ and $\pi(C) = 0$, then $\pi(A \to C) = 0$. This again is not plausible if $\pi(A \to C)$ is the probability of $C \sim A$. Suppose π(*Kennedy was not killed*) = 0 while π(*Oswald killed Kennedy*) = 0.9. Preservation says π(*If Oswald didn't kill Kennedy, he wasn't killed*) = 0. But *If Oswald didn't kill Kennedy, he wasn't killed* read incrementally is *No one other than Oswald killed him.* This is highly probable on anyone's account; it is 81% likely if we are 90% confident that there weren't two killers.

[25] Precis of the next section: If conditional probabilities determine update *counterfactuals* (as they should for the Bayesian), while the probability of the conditional goes with update *dispositions* (as maintained by Stalnaker among others), then THESIS violations are inevitable. A finkish disposition—one whose basis is undermined, or implanted, by its trigger—by definition does not align with the corresponding counterfactual. The disposition constituting acceptance of an incremental conditional is finkish if learning *A* undermines belief in the surplus content. The finkishness of that disposition explains how acceptance of $A \to C$ can happily coexist with low conditional probability of *C* on *A*.

[26] Hall and Hajek call it the *Conditional Construal of Conditional Probability* (CCCP) (Hájek & Hall 1994). Edgington speaks of the *Equation*, because it equates two probabilities, the monadic probability of a conditional and the conditional probability of the consequent on the antecedent.

> To be disposed to accept *C* on learning *A* is to accept *C* conditionally on *A*, or to accept that if *A*, then *B* (Stalnaker (1984), 103)

That leaves (CFL), or counterfactualism. Counterfactualism is perhaps closest to the text. "Adding *A* hypothetically to one's stock of knowledge" sounds a lot like supposing in a counterfactual spirit that *A* is known. "Arguing on that basis about *C*" sounds like arguing about how much confidence, on that counterfactual supposition, to repose in *C*.

Now, for reasons indicated in the last section, THE THESIS has been called into question lately. An example adapted from Kaufmann is this. Suppose that I am to draw a ball from one of two urns, each with a hundred balls in it. The question is, will the ball be shiny, if it is red? Urn 1 contains 10 red balls, 9 of which are shiny, and 90 green balls; Urn 2 contains 90 red balls, 81 of which are dull, and 10 green balls. The urn before me, the one I am reaching into, is 90% likely to be Urn 1. How confident am I that *If the ball is red, it is shiny*? Very confident, because the ball is very likely from Urn 1, and 90% of the red balls in Urn 1 are shiny. The probability of the ball being shiny, conditional on its being red, is

$$\pi(\mathrm{S|R}) = \pi(\mathrm{S|R\&U1})\times\pi(\mathrm{U1|R}) + \pi(\mathrm{S|R\&U2})\times\pi(\mathrm{U2|R})$$

Given that 90% of the red balls in Urn 1 are shiny, $\pi(\mathrm{S|R\&U1}) = .9$ $\pi(\mathrm{S|R\&U2})$ by similar reasoning is .1. We know by Bayes' Theorem that $\pi(\mathrm{U1|R}) = \pi(\mathrm{R|U1})\times\pi(\mathrm{U1})/\pi(\mathrm{R}) = (1/10 \times 9/10) \div 18/100 =$ 50%. $\pi(\mathrm{U2|R})$ will be 50% too—the ball must come from one urn or the other—so

$$\pi(\mathrm{S|R}) = (.9 \times 50\%) + (.1 \times 50\%) = 50\%$$

If the probability of drawing a shiny ball, conditional on its being red, is 50%, then $\pi(S|R)$ is a far sight short of $\pi(R\rightarrow S)$, which was seen to be high (82%). The THESIS thus appears to be off by a factor of over half (32/50) in equating $\pi(R\rightarrow S)$ with $\pi(\mathrm{S|R})$.

What about (CFL)? Our agent is a Bayesian, let's assume, who updates by conditional credence.[27] Her confidence in *S*, when she learns that *R*, is her *prior* confidence in *S* given *R*. But then we run into the same sort of problem as before. The confidence she *would* have in the ball's being shiny, on learning that it was red, is *low*, for she found it antecedently unlikely that a ball would be shiny, given that it was red. But although the counterfactual *Had she learned the ball was red, she would have concluded it was shiny* is mistaken, she is 82% sure that the (still undrawn) ball is shiny if red.

[27] $\pi_E(A) = \pi(A|E)$.

If the counterfactual account falls to Kaufmann examples, then the dispositional acount might seem vulnerable too—for the disposition on *x*'s part to do such and such when suitably triggered lines up in many cases with the truth of the corresponding counterfactual: *x would* do such and such *were* it triggered. Stalnaker explicitly distances himself from this idea. (DIS) is not meant to imply "that the rational agent who accepts *If A then C* will always come to accept *C* on learning *A*" (104)—for the learning experience may erode his confidence in the conditional. There are at least two ways this could come about. He may wind up learning *more* than *A*, where the totality of what is learnt argues against *C*. I do not know about it, if my friend is lying to me; he is pretty convincing. Would I accept the consequent—that I don't know about his lying—on learning (just) that he is lying to me? The question is hard to make sense of, since I would almost certainly learn, in addition, that I was learning it, which ruins the experiment.

Also though, the new knowledge may erode one's confidence in the conditional even if one learns exactly *A*. I am thinking here of what used to be known as *the conditional fallacy*.[28] An undropped vase is still fragile, even if its guardian angel has resolved to surround it in bubble wrap, or tinker with its molecular structure, should it be dropped. The disposition would not be destroyed by this decision but only *masked*; one should not expect a disposition to manifest itself counterfactually if the trigger would undermine its basis. Going in the other direction, an undropped styrofoam cup does not become fragile when an avenging angel decides that she would fragilize its underlying structure were it dropped. The disposition in this case is *mimicked* but not really there.

Dispositions to believe are like any other dispositions in this respect. I would indeed not conclude the ball was shiny, on learning that it was red. But that does not show I lack the *disposition* to pronounce it shiny on discovering it to be red. The disposition would not be manifested, because it would not survive that discovery.[29] This is a mimicking-style counterexample to THE THESIS, and to counterfactualism, but not as far as I can see to dispositionalism.

How far are these subtleties reflected in the probabilities? That I would expect the ball to be quite possibly dull is predicted by my conditional probabilities, not the probability I assign to *If the ball is red, it will be dull*. I assign a low probability to that conditional and lack the

[28] Shope (1978)

[29] Huw Price makes a related suggestion about conditional probability itself, based on a remark of Ramsey's: "The degree of belief in *p* given *q* is not the same as the degree to which a subject would believe *p*, if he believed *q* for certain; for knowledge of *q* might for psychological reasons profoundly alter his whole system of beliefs" (Ramsey (1931), 180). Ramsey is talking about *irrational* update behavior, however. Conditional probability *does* predict the update behavior of rational Bayesians with respect to *q*. Update behavior is not predicted, I'm saying, even for the rational Bayesian, by the prior probability of *If q then p*, because there is nothing irrational in losing confidence in *If q then p* on learning that *q*.

update disposition as well, even if I would acquire it when I learned the antecedent. That I would not expect the ball to be shiny is again predicted by my conditional probabilities, not the probability I assign to *If the ball is red, then it is shiny*. I assign a high probability to that conditional and possess the update disposition as well, even if I would lose it when I learned the antecedent. The outcome in either case is this, where $\alpha > \gamma$ means that the agent would believe C if she were to learn that A, and $\alpha \gg \gamma$ means that she is *disposed* to believe C on learning that A: we shouldn't *expect* the THESIS to hold in full generality if $\pi(C|A)$ lines up with $\alpha > \gamma$ (an update counterfactual), while $\pi(A \rightarrow C)$ lines up with $\alpha \gg \gamma$ (an update disposition), for counterfactuals don't always line up with dispositions. This is indicated in the diagram by the "¬iff" between $\alpha > \gamma$ and $\alpha \gg \gamma$ on the left, and the corresponding "¬iff" on the right between $\pi(C|A)\approx 1$ and $\pi(A\rightarrow C)\approx 1$ (Figure 1).

$$\begin{array}{ccc} \alpha > \gamma & \overset{\text{iff}}{\longleftrightarrow} & \pi(C|A) \approx 1 \\ {\scriptstyle \neg\text{iff}}\uparrow & & \uparrow{\scriptstyle \neg\text{iff}} \\ \alpha \gg \gamma & \overset{\text{iff}}{\longleftrightarrow} & \pi(A \rightarrow C) \approx 1 \end{array}$$

Figure 1: Conditionals' probabilities stand to conditional probabilities as update dispositions stand to update counterfactuals.

When they do line up—when the antecedent is probabilistically independent of the facts thought to underly the disposition—the THESIS holds. It fails when, as in the case of *If red then shiny*, the antecedent disconfirms the underlying assumptions, or, as in the case of *If red then dull*, it makes them for the first time believable.

Now, the truth of $A \rightarrow C$, read absolutely, is grounded in the fact that Δ^A_C.[30] Suppose with Stalnaker that acceptance of $A \rightarrow C$ is something like a disposition to believe that C on acquiring the belief that A. It stands to reason that this should be a *grounded* disposition underwritten by *belief* in Δ^A_C. This makes for a neat overall package in which *the fact making* $A \rightarrow C$ *true* = *the fact belief in which makes* $A \rightarrow C$ *acceptable* (Figure 2).

The parallel depicted here, between the grounding of conditional truths in Δ^A_C and of the corresponding acceptance-states in awareness of Δ^A_C, has consequences both for the logic of absolute conditionals and their epistemology. The one we'll eventually be focusing on (Section 12) is that $A \rightarrow C$ *cannot be settled just by asking how many A-worlds verify* C; for Δ^A_C cannot be settled in that fashion.[31]

[30] Δ^A_C is a different and possibly more suggestive notation for $C \sim A$, the "difference" between A and C.

[31] Precis of the next section: THESIS failures line up quite often with a nonstandard way of calculating conditional probabilities, recently emphasized by Kaufmann. There are two possible states of the world, H and $\bar{H}$; C's probability conditional on A is one thing in the H-region, another in the $\bar{H}$-region; we take the average of those two conditional probabilities, weighted by the likelihood, respectively, of H and $\bar{H}$.

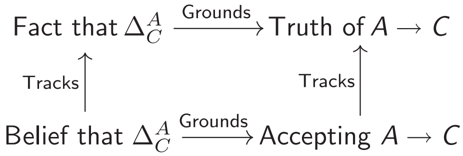


Figure 2: A fact Δ^A_C grounds the truth of *A*→*C*; belief in Δ^A_C grounds (the disposition constituting) belief in *A*→*C*.

VIII. Local/Global

The probability of *A*→*C* is not in general the probability of *C* conditional on *A*. Examples have been given in the last two sections. The probability of *#1 says Mrs Hudson did it* → *She did it* is a great deal higher than the probability of *She did it* conditional on *#1 says she did it*. Where do we get our confidence in *#1 says she did it* → *She did it* if not from the conditional probability? We seem to reason like this (following Kaufmann 2004): There are two possible states of the world: #1 is Holmes or #1 is Watson. Since the probability that Mrs Hudson did it conditional on #1 saying so is quite different, depending on who #1 is, we adopt the following strategy.

Step One: Divide logical space between the *#1 is Holmes* scenario and the *#1 is Watson* scenario.
Step Two: Estimate the probability of *Mrs Hudson did it* conditional on *#1 accused her* once for each scenario.
Step Three: Obtain the overall probability as a function of these two local conditional probabilities.

How likely is it that Mrs Hudson did it, conditional on *Holmes* accusing her? Highly likely, say 99%. How likely is it that Mrs Hudson did it, conditional on *Watson* accusing her? Not likely at all, maybe 2%

π(*Hudson did it* | *#1 says it was Hudson* & *#1 is Holmes*) = 0.99
π(*Hudson did it* | *#1 says it was Hudson* & *#1 is Watson*) = 0.02

So should we just take the average of .99 and .02? Certainly not. That would be to treat both hypotheses about the identity of *#1* as equally probable, when the Holmes hypothesis is much better supported. We want rather the *weighted* average, where .99 and .02 are multiplied, respectively, by the prior probability of *#1 is Holmes* (= 90%) and *#1 is Watson* (= 10%). Our credence that *If #1 says it was Mrs Hudson, then it was Mrs Hudson* is

$\pi(M|\text{'}M\text{'}\&H)\times\pi(H) + \pi(M|\text{'}M\text{'}\&W)\times\pi(W)$
$= (.9\times.99) + (.02\times.01) =$ about 90%.

This conditional, evaluated by playing off two hypotheses about the (relatively) objective set-up that controls conditional chance, has come to be called the *local* conditional. The idea behind it—the idea of treating certain conditionals as rational guesstimates of "real", or at least better informed, conditional probability—goes back to earliest days.[32] The fact is that we are inclined to speculate on the underlying circumstances that make C more or less likely given A.[33] Let the hypotheses between which we're undecided be the X_is, and the set of them (the partition) be X. Then the localized conditional probability is

$$\text{LOCAL: } \pi_X(C|A) = \Sigma_i\ \pi(C|A\&X_i) \times \pi(X_i).$$

[32] Adams considers a counterfactual analogue in Chapter 4 of *The Logic of Conditionals* (1970). Here is Edgington's version of Adams' (example Edgington (2014)):

John has a rare disease. There are two drugs, D and E, which would help him. If he takes just one, he'll get better. If he takes both or neither, he'll get worse (though the harmful effect of taking both is not well known...). Both are in extremely short supply, and it's very unlikely that he'll get either, but it's about 100 times less likely that he'll get E than that he'll get D, and immensely unlikely that he'll get both. I think *If John takes E, he'll get better.* Now John does get better. Much the most likely explanation is that he got D. So now I think *If he had taken E he would have got worse.*

In thinking that John would have got worse if he'd taken E, I am *disagreeing*, and in a foreseeable way, with my earlier judgment that *If John takes E, he'll get better.* Reflection suggests that the earlier judgment was mistaken, or that it has a reading on which it was mistaken. But that original judgment is what you'd expect given my prior conditional probabilities. The prior probability of John getting better, given that he takes E, is high, say 99%. And yet *If John takes E, he will get better* was, given that John does get better, probably in fact false—and we have as yet no explanation of that. Adams suggests one:

A rather simple minded generalization of our prior conditional probability representation [$\pi(B|E)$] which would accommodate [this] is as follows.... we may suppose that there are mutually exclusive and exhaustive states D and $\neg D$ which are causally independent of whether John takes E,.... In this case the probability of the [conditional] is plausibly given by $\pi(E\rightarrow B) = \pi(B|D\&E)\times\pi(D) + \pi(B|\neg D\&E)\times\pi(\neg D)$ (Adams (1975))

The chances are (since John got better) that he has taken D. So the first term in the expression for $\pi(E\rightarrow B)$ dominates. $\pi(B|D\&E)$ is low since the two drugs together will make John worse. This is why our confidence in $E\rightarrow B$ declines after learning that John gets better. The thought more generally is that, rather than using a simple conditional probability $\pi(C|A)$, we take the weighted sum of conditional probabilities as we vary the underlying probability-determining facts. (Adams only toys with the local rule as he attempts to deal with a certain puzzle. The rule he prefers is the one that figures in the THESIS: $\pi(E\rightarrow B) = \pi(B|E)$).

[33] Skyrms calls these hypothesis-driven probabilities "propensities."

This agrees in some cases, but not all, with the global conditional probability $\pi(C|A)$. Standard rules allow us to rewrite that global probability as

$$\text{GLOBAL: } \pi(C|A) = \Sigma_i\ \pi(\mathrm{C}|A\&X_i) \times \pi(X_i|A).$$

The two formulae come to the same if the antececent A is probabilistically independent of the issue of which hypothesis in fact obtains – if $\pi(X_i|A)$ is the same as $\pi(X_i)$. This is not so in the Holmes case, because whether #1 accuses Mrs Hudson is not probabilistically independent of whether #1 is Holmes. It is not so in the Urn example, because whether the ball is red is not probabilistically independent of which urn it came from. Or consider an example of Roger White's: *If Brad's book (advocating 4-dimensionalism) is correct, then nothing changes.* Whether a book is correct is not independent of whether it says that nothing changes, and so not independent of whether *If Brad's book is correct, then nothing changes.*[34]

IX. Buts

Where does this leave us? The plan was to try, in an experimental spirit, to build our theory around the conditionals we called *absolute.* This led to surplus content, otherwise known as what C adds to A. Surplus content, we decided, could be characterized in terms of enthymemes; it's the R that slots perfectly into the enthymeme $A, ???\therefore C$. Part of slotting perfectly in is taking full advantage of A. "Takes full advantage" is only a metaphor, of course. I propose to turn now to the rest of the title—"...Ands and Buts"— in the hope that this may help us to unpack the metaphor.

The usual view among philosophers is that "but" implies "and," adding to it only some kind of pragmatic hint that the second conjunct is somehow unexpected given the first. This is crude (Kripke (2011), Toosarvandani (2014)) but not in a way that matters here. For we are interested in a kind of "but" that does not meet even that minimal requirement—that works in fact *against* conjunction. You see it in sentences like *Every Justice asked a question but Thomas. Every Justice asked a question but Thomas* does not say that every Justice asked a question *and* Thomas asked one. For one thing the conjunct would be redundant; to say that all of the Justices asked questions already implies that Thomas did. Does the speaker perhaps want to drive that implication

[34] To be clear, Brad's book does not in fact say that nothing changes. Precis of the next section: Surplus contents have not yet been fully explained. Inspiration is sought in "exceptive constructions" like *I agree, but for the bit about China.* A binary connective $\sim$ is introduced on a model of "but for." The new connective is an inverse to conjunction insofar as $(X\sim Y)\&Y$ is equivalent to X.

home? Adding "but Thomas" achieves the exact opposite. It functions intuitively to *cancel* any suggestion that Thomas asked a question.[35] When "but" is used to carve out exceptions, it expresses something like the *inverse* of conjunction; see below. *Everyone asked a question but Thomas* falls short of *Everyone asked a question* precisely by Thomas answering a question; its truth-conditional content is obtained by subtracting *Thomas asked a question* from *Everyone did.*

The claim so far is that buts are, or can be, exception-makers along the lines of "present company excepted" or "leaving aside the typos" or "bracketing any requirements still under review."[36] The usual examples involve quantification. But I will understand the class more broadly than this, allowing buts in their exceptive sense to function as main connective. This subsumes the quantificational use for a reason already mentioned. *Everyone but Thomas asked a question* says that *Everyone asked a question, apart from the bit about Thomas asking one.*

So, we are interested in a logical connective $P \sim Q$ whereby, if P implies Q, that implication is cancelled or waived or stripped away.[37] We can think of $\sim$ as undoing the effect of conjunction, or better, as the operator such that conjunction undoes *its* effect: $(P \sim Q) \& Q$ is true in the same worlds as P.[38] I'm not sure there is a perfectly colloquial way to formulate these kinds of statements, but the following are in the ballpark: *P, except maybe not Q*; *P, apart from the bit about Q*; and *P, but for Q.*

1. Every Justice asked a question, with the possible exception of Thomas.
2. Kennedy was killed by someone other than Oswald, or for that matter by Oswald.
3. The king is in the counting house, though don't hold me to that Nigel guy really being the king.
4. These two triangles are congruent, waiving the requirement of being the same size.
5. Pete won, ignoring the possibility that he folded.

What do these statements say? How is the proposition to be identified? The arithmetical analogy floated above may be helpful. Consider the problem of identifying m minus n on the basis of m and n. This is solved, we know, by looking for the unique number r such that adding r to n gets you back to m. An analogous proposal about $C \sim A$ would be this: it's the R that has to be added to A to get C.

[35] Some may think it adds the opposite implication. I will assume not. Any suggestion that Thomas in fact *didn't* ask a question will be treated as an implicature.

[36] Gajewski (2008), Von Fintel (1993).

[37] The operator still applies if P does not imply Q, only its action cannot be described in this case as cancelling an implication.

[38] $(P\&Q) \sim Q$ is trivial if $P = Q$; it would be P if $\sim$ just undid the effect of conjunction.

The problem is that there might be any number of *R*s such that *A* and *R* imply *C*, ranging from $A \supset C$ to $A \equiv C$ to *C* itself. Some of these *R*'s are, once again, better than others. The *R* we want is the hypothesis that completes the enthymeme *A*, *???*$\therefore$*C* in the most *A*-beholden way. How on this way of doing things are we to find *Every Justice asked a question* ~ *Justice Thomas asked a question*? First let's set out the relevant enthymeme.

Thomas asked a question.
⇒ ???
Every Justice asked a question.

To complete this in a maximally *A*-beholden way, we look for the feature of certain *Thomas asked a question*-worlds whereby they are *Every Justice asked a question*-worlds. That feature is, I take it, that every other Justice asked a question in them. Again, what is the shortest path from *Nigel is the king* to *The king is in the counting house*? We look again for the feature of certain *Nigel is the king*-worlds that makes them *The king is in the counting house*-worlds. Nigel is in the counting house in those worlds. *The king is in the counting house* ~ *Nigel is the king* is therefore *Nigel is in the counting house*. Let us try finally to strip *Pete won* of the implication that Pete called. What is going on in certain *Pete called*-worlds to make them *Pete won*-worlds? Well, Pete had the better hand in those *Pete called*-worlds. *Pete won* ~ *Pete called* is therefore *Pete had the better hand*. A more precise statement, in terms of truthmakers, is given in Section 11.[39]

X. Affinities

Remainders and conditionals have a good deal in common. Remainders are apt indeed to be *formulated* in conditional terms without anyone noticing or remarking on the fact. Consider the view called ifthenism in the philosophy of mathematics. One account sometimes heard of this view is that a statement like

> *The number of stars is finite* is used to express, not its full number-involving content, but what that content adds to the assumption that there are numbers in the first place—presumably that there are finitely many stars.

A second, equally common, explanation says that

> *The number of stars is finite* is used to express, not its literal content, but that content conditioned on the assumption of numbers, viz.

[39] Precis of the next section: Remainders *C* ~ *A* and conditionals have a number of affinities, ranging from the language used to express them to the graphical representations that suggest themselves when we look for their truth-conditions.

that the number of stars is finite, if (or on the assumption that) there are numbers.

The label if/thenism puts us in mind of the second, conditional, formulation—which is odd, because the conditional formulation is risky; it leaves room for the (unwanted) thought that the existence of numbers might be *evidentially* relevant to how many stars there are. (An oracle might have told you that more stars means fewer abstract objects, and vice versa.)[40] The worry about evidential relevance strikes us as ridiculous and misconceived. That it *is* ridiculous if the conditional is remainder-like suggests that if/thenists are implicitly understanding *If the number of stars exists, then it is finite,* as what the consequent adds to the antecedent, viz. that *There are finitely many stars.*

A second affinity is to do with the phenomenon of non-catastrophic presupposition failure. It has long been recognized that while the falsity of a presupposition *P* may *sometimes* make a sentence unevaluable —as *The king of France is bald* seems unevaluable—there are plenty of such sentences that strike us as false: *The king of France is bald and pigs fly*, for instance, and *The king of France is sitting in this chair. S* will strike us as false, despite the failure of its presupposition *P,* if *S* adds something to *P*—*Pigs fly, The chair is not empty*—which is independently evaluable and turns out to be false, that is, if $S \sim P$ is false. $S \sim P$ is a remainder, obviously, but one is strongly tempted to reformulate it as a conditional (Lasersohn 1993):

Even if France has a king, still, it is not the case that: He is bald & pigs fly. Even if France has a king, still, he is not sitting in this chair.

These too are liable to be misunderstood. Imagine someone convinces us, never mind how, that the king of France is a master illusionist who is sitting undetectably in that chair, if he exists. If *The king of France is sitting in that chair* seemed false before, it continues to now. The appearance of falsity is based on the fact that the chair is empty. Whether it is still empty on the hypothesis that France has a king is irrelevant; all that matters is that the sentence adds something false to its presupposition. This is just to say that the reformulated test is attractive only insofar as we are reading the conditionals incrementally. Clearly it *is* attractive; so that must be how we are reading them.

The rewriteability point extends as well to the remainders mentioned above—*Every Justice with the possible exception of Thomas asked a question, The king is in the counting house, albeit Nigel might not really be king*. The corresponding conditionals are

[40] Horgan's counterfactual style of if-thenism runs into an analogous problem (Horgan 1984). Hellman suggests a "non-interference proviso": "we must stipulate from the outset that the only possibilities we entertain in employing the [modal operator] are such as to leave the actual world entirely intact." (Hellman 1989:99).

1. If Thomas asked a question, all the Justices asked questions.
2. If Oswald didn't shoot Kennedy, then someone else did.
3. If Nigel is the king, then the king is in the counting house.
4. If these triangles are the same size, they're congruent.
5. If Pete called, he won.

These conditionals seem in each case to admit a non-evidential interpretation whereby they stand or fall with $C \sim A$. The Oswald conditional is decided by whether Kennedy was killed. The Thomas conditional is decided by whether everyone other than Thomas asked a question. The Nigel conditional is decided by whether Nigel is in the counting house. The Pete conditional is decided by whether Pete had the winning hand. That all the conditionals can be read incrementally is the third point of analogy between "if" and "but."

A fourth affinity is that $A \rightarrow C$ and $C \sim A$ lend themselves to a similar sort of graphical representation (see below). Both require us to extrapolate in some way from the distribution of C in "home"-worlds (A-worlds) to its likely distribution in "away"-worlds ($\neg A$-worlds). They both agree with C in the home-region. The challenge with conditionals and remainders alike is to find a way to model their away behavior on their behavior at home.

Doubts are often expressed about the possibility of carrying out this extrapolation in a principled way. What are we to think of *If Cora is in France, she's in Lyon* in a world where Cora is in Antarctica?[41] How is one to extend the principle behind *Cora's car is dark red* to worlds where the car is green? What does *I raised my arm* ask of worlds where my arm does not go up? (To say nothing of extending *It's five o'clock here* to worlds where "here" is on the Sun, or altogether outside of the solar system.[42]) To gather the structural affinities between $A \rightarrow C$ and $C \sim A$ into one place,

Both are implied by $A \& C$.
Both are inconsistent with $A \& \neg C$.[43]
Both attempt to extend C from "home" to "away".
Both extensions are apt to be problematic.
Neither extension is *inherently* problematic.[44]

These analogies are what license us in postulating a conditional connective $\rightarrow_\alpha$ understood so that $A \rightarrow_\alpha C$ holds (i) in the same worlds as $C \sim A$, (ii) on account of their being $C \sim A$-worlds.

Given the connection with $\sim$, a better notation for this conditional might be $\rightsquigarrow$; this is what I will use henceforth. *Pete called* $\rightsquigarrow$ *He won* holds where, and because, Pete had the better hand. *Nigel is the king* $\rightsquigarrow$ *The*

[41] Skepticism about the away-behavior of conditionals is expressed in Belnap (1970), Quine (1982), McDermott (1996), and Cantwell (2008).

[42] See, for doubts about the away behavior of remainders, Jaeger (1973) and Jackson (1977).

[43] Both are intermediate in strength, then, between $A \& C$ and $A \supset C$.

[44] $A \& B \sim A$ is in many cases just B.

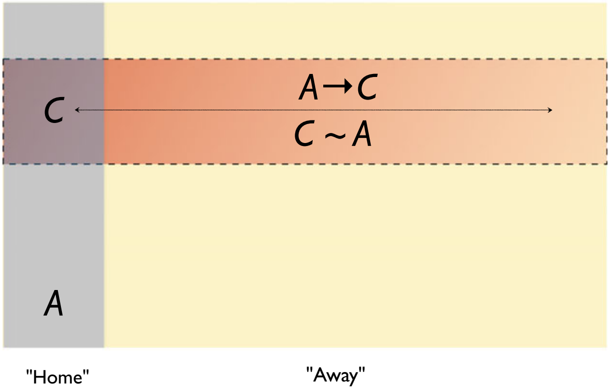


Figure 3: Conditionals and remainders represented graphically.

king is in the counting house holds where Nigel is in the counting house, *because* of his being there. The claim is that we have in $A \rightsquigarrow C$ a prime contender for the role of absolute conditional. In support of this, if one looks for an extrapolation strategy suited to $C \sim A$, one winds up with something like the suggestion above that $A \rightarrow C$ holds absolutely in w if a fact X obtains in w that explains why w would be C given that it is A. This is explored in the next section.[45]

XI. Extrapolation

How in graphical terms are we to solve "$C \sim A = R$" for R? R has been explained informally as the result of extending C's behavior at home, where A holds, to the away region, where A is false. From this it seems that R's truth-value in the away region ought to be controlled by the same sorts of factors as distinguish $A\&C$-worlds from $A\&\neg C$-worlds. The proposal is again going to involve truthmakers for $A \supset C$. But we do it more carefully now, with an eye especially to making the most of A.

Looking back at Figure 3, we can see that extrapolation is going to require two types of rule. "Home" rules speak to R's behavior at home, that is, within the A-region. An R that continues C beyond that region should at a minimum follow C's lead within it. "Away" rules speak to R's behavior *outside* the A-region. If R is to divide up away-worlds on the same principle as at home, its away behavior should agree in some still undetermined way with its behavior at home. A second, cross-cutting, distinction is between "classifying" conditions, which are to do with *whether* R is true in a given world, and "rationalizing" conditions which are to do with *how and why* R is true in a world. This gives us four types of condition overall—home-classifying, home-

[45] Precis of the next section: $C \sim A$ attempts to extend C's behavior at home, where A holds, to the away region, where A is false. Rules are suggested for carrying this extension out.

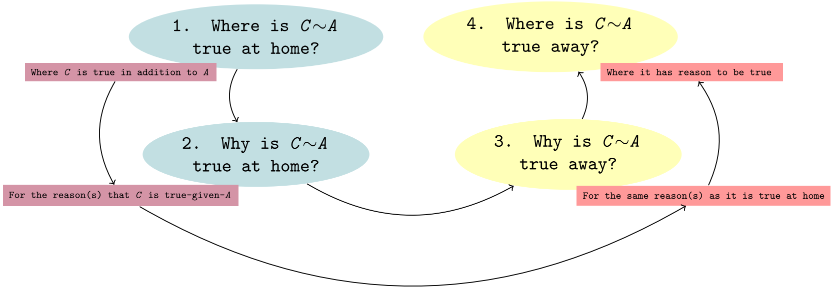


Figure 4: Extrapolation in four easy steps.

rationalizing, away-classifying, and away-rationalizing. The following order seems logical: 1. HC, 2. HR, 3. AR, 4. AC. The flow chart shows how the construction will have to go.

So, let us try it, beginning at home and extending patterns established there into the away-region. R = "what C adds to A" should have the same truth-value as C in the A-region. It should be true if $C\&A$, and false if $\neg C\&A$. Because the requirement on R is to agree truth-value-wise with C at home, we call this condition

***Agreement* (1):**

$$R \text{ is } \left\{\begin{matrix} \textit{true} \\ \textit{false} \end{matrix}\right\} \text{ at home in the same worlds as } C \text{ is } \left\{\begin{matrix} \textit{true} \\ \textit{false} \end{matrix}\right\}.$$

Next, given that a home-world w is R, how does it acquire that status? To answer a question with a question, why is w on the $A\&C$ side of the line rather than the $A\&\overline{C}$ side? (Why is it a *The no. of dragons = 0* world rather than one where *The no. of dragons* $\neq$ 0)? *That* is why R holds true in w. R's reason for being true (false) in a home-world w is whatever makes C true (false) in w, *given that A is true there.*[46] (Whatever that means—see below.)

***Reasons* (2):**

$$R \text{ is } \left\{\begin{matrix} \textit{true} \\ \textit{false} \end{matrix}\right\} \text{ at home for the same reason(s) as } C \text{ is } \left\{\begin{matrix} \textit{true} \\ \textit{false} \end{matrix}\right\} \textit{given } A.$$

Next we have to specify R's ways of being true/false in *away* worlds. A hypothesis "goes on in the same way" from the home-region if its truth is controlled by the same factors. R should not acquire new truthmakers and falsemakers when it leaves home.

***Integrity* (3):**

$$R \text{ is } \left\{\begin{matrix} \textit{true} \\ \textit{false} \end{matrix}\right\} \text{ for the same reasons away as it was } \left\{\begin{matrix} \textit{true} \\ \textit{false} \end{matrix}\right\} \text{ at home.}$$

[46] In this case: It doesn't have any dragons.

R's truth-value(s) in away-worlds are a function, finally, of the available *reasons* for R to be true/false in such worlds.

Projection **(4):**

$$R \text{ is } \left\{ \begin{array}{c} \textit{true} \\ \textit{false} \end{array} \right\} \text{ in an away-world } w \text{ just if it has reason to be } \left\{ \begin{array}{c} \textit{true} \\ \textit{false} \end{array} \right\} \text{ in } w.$$

Reasons could stand to be clarified. What are C's reasons for being true *given* A, in an A-world? A reason for C to be true-given-A is a truthmaker X for $A \supset C$ that is consistent with A and makes the fullest possible use of A. X does that, the claim is, *if it minimizes the extent to which* $B \supset C$ *is also implied*, for B weaker than A. The official definition is in three steps. A^- in the first step ranges over hypotheses weaker than—asymmetrically implied by—A.[47]

1. X' *uses more of* A than X if $\{A^- | A^-, X' \nvDash C\} \subsetneqq \{A^- | A^-, X \nvDash C\}$
2. X is *wasteful in* w if an X' holding in w uses more of A than X does.
3. X is *wasteful* (period) if it is wasteful in every A-world where it holds.

A truthmaker for $A \supset C$ is *targeted*, finally, if it's compatible with A—it doesn't falsify the antecedent—and doesn't waste A—with the result in most cases that it doesn't imply the consequent.[48] Truthmakers of this sort will also be called *difference*-makers as between the antecedent and the consequent.

> $C \sim A$ is true (false) in w iff $A \supset C$ ($A \supset \neg C$) has a targeted truthmaker in w and $A \supset \neg C$ ($A \supset C$) doesn't, that is, a difference-maker obtains in w for A and C, but not A and $\neg C$.

This agrees, more or less, with the truth-conditions given earlier for absolute conditionals; $A \rightarrow_{\alpha} C$ is true in w, we said, iff a fact obtains there that explains why a world stipulated to be A would be moreover C.[49]

XII. Conservativeness

Recall the notion of *conservativeness* from generalized quantifier theory. Predicates G and H are *F-equivalent* iff $F\&G$ is equivalent to $F\&H$,

[47] Alternatively we could let it range over A's parts; this paper is not about content-parts, however, so we use the definition in the main text.

[48] Targeted truthmakers should take advantage of A, unless there is no advantage there to be taken. Then they have no option but to imply C. This occurs, for instance, with *If the balloon is colored, it's red* and *If it's not red, it's nevertheless colored.*

[49] Precis of the next section: Existing theories make indicative conditionals *conservative* in the sense that consequents C and D are freely substitutable provided only that they agree on A-worlds. This seems like the wrong prediction both for marketplace conditionals and incremental ones. The latter have the advantage of being non-conservative for clear, understandable reasons.

that is, if all and only *G*s are *H*'s, when we restrict attention to the *F*s. A quantifier $\mathcal{Q}$ is conservative iff

If *G* and *H* are *F*-equivalent, then $\mathcal{Q}$(*Fs are Gs*) iff $\mathcal{Q}$(*Fs are Hs*)

So for instance, *ALL* is conservative because whether all dogs are friendly turns entirely on what goes on with the dogs; which non-dogs might be friendly is completely irrelevant. *SOME* is conservative because whether some dogs are friendly again turns entirely on what goes on with the dogs; the non-dogs you should feel free to ignore. *MOST* is conservative because whether most dogs are friendly is determined by the proportion of friendly dogs to non-friendly dogs; it doesn't matter how friendliness is distributed among cats. Conservativeness is sometimes thought to be a semantic universal for quantifiers; no natural language quantifiers care in the slightest about things that are not *F*.

The corresponding property for conditionals $A \to C$ is that if the same *A*-worlds are *C* as *D*, then $A \to C$ is equivalent to $A \to D$. Using *A*-equivalence for the property of equivalence over the *A*-worlds,

If *C* and *D* are *A*-equivalent, then $A \to C$ is true (correct, probable,....) iff $A \to D$ is true (correct, probable,....).

(The two kinds of conservativeness are related insofar as the truth- or correctness conditions of conditionals are given in quantificational terms, which in practice they generally are:

all *A*-worlds are *C*-worlds
most *A*-worlds are *C*-worlds
the closest *A*-worlds are *C*-worlds.)

Existing theories, their other differences notwithstanding, somehow wind up agreeing on this one point. They all make if/then out to be *conservative*: it is only *C*'s overlap with *A* that matters to the correctness of $A \to C$. The reasons vary, of course. $A \to C$ holds, according to Goodman, Adams, Kratzer, Stalnaker, etc., just if

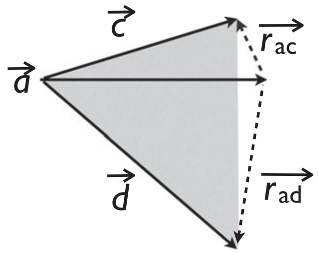
truths cotenable with *A* entail $A \supset C$ (Goodman)
....which is the same as cotenable truths entailing $A \supset (A \& C)$

C is highly probable, conditional on *A* (Adams)
.....which is the same as $A \& C$ being highly probable conditional on *A*

C holds in all (the best) *A*-worlds (Kratzer)
.... which is the same as $A \& C$ holding in all (the best) *A*-worlds

C holds in the closest *A*-world in the context set (Stalnaker)
... which is the same as $A \& C$ holding in that world.

C is probable given *A* for every π in the context set (Yalcin)
... which is the same as $A \& C$ being conditionally probable for every π



Obliviousness to C's away behavior is not a problem, obviously, if away-behavior does not matter. First an analogy to show how it *could* matter. The analogy is with vector subtraction. That $\vec{c}$ and $\vec{d}$ agree in how they project onto $\vec{a}$ does not mean that $\vec{c}-\vec{a} = \vec{d}-\vec{a}$ ($\vec{r}_{ac} = \vec{r}_{ad}$).

Conservativeness is implicated in a number of well known puzzles. Sly Pete examples, for instance, in which pairs of conditionals $A \rightarrow C$ and $A \rightarrow \neg C$ both seem right, are hard to handle if $\rightarrow$ is conservative, since A-worlds cannot be both C and $\neg C$. Agglomeration failures, in which $A \rightarrow C$ and $A \rightarrow D$ do not imply $A \rightarrow (C\&D)$, should be impossible; but *If Bizet and Verdi are the same height, Verdi is short* and *If Bizet and Verdi are the same height, Bizet is tall* do not seem to imply *If Bizet and Verdi are the same height, Verdi is short and Bizet is tall.* And there are just straight-up intuitive counterexamples: *If Oswald didn't kill Kennedy, someone else killed him* ought if conditionals are conservative to come out equivalent to *If Oswald didn't kill Kennedy, Kennedy was killed.* Conservatives have their responses, of course! But responses would not be needed if a non-conservative (progressive) alternative could be made out.[50]

XIII. Incremental Progress

Incremental conditionals are away-dependent, or progressive, right out of the box. Just as the $\vec{\mathbf{r}}$ that gets you from $\vec{\mathbf{a}}$ to $\vec{\mathbf{c}}$ may not fit snugly into the gap between $\vec{\mathbf{a}}$ and $\vec{\mathbf{d}}$, the shortest path from A to C may not be the shortest path from A to D.[51] Incremental conditionals

[50] Precis of next section: Examples are given of intuitively inequivalent conditionals $A \rightarrow C$ and $A \rightarrow D$ that should on conservative principles be equivalent, since their consequents C and D are true in the same A-worlds.

[51] If this is true, it ought to show up somehow in our extrapolation rules. Look again at ***Reasons***: R is true (false) in an A-world w for the same reasons as C is true (false) there *given A*. Suppose that D holds in the same A-worlds as C. Must it hold, given A, for the same reasons? Not at all. *Your brother is an idiot* holds, given that that guy is your brother, because *that* guy is an idiot. Is it also because that guy is an idiot that *That guy is an idiot* holds, *given* that that guy is your brother? No, that may be why *That guy is an idiot* holds simpliciter, but its reason for holding *given* that that guy is your brother is different. The reason D holds *given A* is a targeted truthmaker for $A \supset D$; the fact that D is not targeted, because it wastes the antecedent.

Table 1

Away-dependent Conditionals; C and D Agree in A-words but A→C is True and A→D is False

Antecedents are Identical		Consequents Agree at Home	Conditionals Disagree Away from Home
IF A. that guy is Smith's murderer,	THEN	C. Smith's murderer is insane. D. that guy is insane.	TRUE cuz: That guy is insane. FALSE cuz: Smith's murderer is insane
IF A. my X-fibers were firing,	THEN	C. I was anxious. D. the theory is right.	TRUE cuz: The theory says so. FALSE cuz: I was not feeling anxious.
IF A. Bizet and Verdi are the same height,	THEN	C. Verdi is short. D. Bizet is short.	TRUE cuz: Bizet is short. FALSE cuz: Verdi is tall.
IF A. This book is correct,	THEN	C. nothing ever changes. D. it says nothing ever changes.	TRUE cuz: That's what the book says ... FALSE cuz: Things change.
IF A. Pete called,	THEN	C. he won. D. his hand was better.	TRUE cuz: Pete had the better hand. FALSE cuz: Pete didn't know both hands.
IF A. Pete won,	THEN	C. Mr. Stone lost. D. he was playing Mr. Stone.	TRUE cuz: Pete was playing Mr. Stone. FALSE cuz: He cannot beat Mr. Stone.
IF A. loving you is wrong,	THEN	C. I don't want to be right. D. I want to love you.	TRUE cuz: I can't give you up. FALSE cuz: I want to be right
IF A. that's your best,	THEN	C. I want better than your best. D. I want better than that.	TRUE cuz: I want better than that. FALSE cuz: I don't want the impossible.
IF A. Hettie sits on those eggs,	THEN	C. they will hatch. D. they're viable.	TRUE cuz: They are viable. FALSE cuz: She is a bad judge of viability.

are not conservative because their truthmakers are supposed to pick up where *A* leaves off, and yet take us to different destinations (*C* and *D*). Marketplace conditionals, too, show a preference for picking up where *A* leaves off, however. Here is a string of examples that hopefully bear this out. $A \rightarrow C$ (in the first column of Table 1) is predicted by standard theories to stand or fall with $A \rightarrow D$ (in the second). The third column indicates how their truth-values can come apart.

A bit of commentary may be helpful. The first, Donnellan-style, example, ought to be familiar. The structure is this: *If a = b, then Fa* says, if we make most of the antecedent, that *Fb*, while *If a = b, then Fb* says that *Fa*. *Fa* can come apart truth-value-wise from *Fb* in worlds where *a* isn't *b*.

The second comes from the philosophy of mind. It is agreed, let us say, that brain states necessitate mental states. But there is controversy about which necessitate which. Our currently best psychophysical theory, theory T, says that X-fiber firings necessitate anxious feelings. I have no idea what my brain state was at noon, but I take the theory's word for it that *If my X-fibers were firing at noon, I felt anxious then.* I am aware of course that I have other ways of determining whether I was anxious at noon, like trying to remember. Suppose I remember being anxious. Then the theory was right about the mental state corresponding to X-fiber firings, if mine were firing at noon. Suppose on the other hand that I was not anxious. Then the theory was wrong if mine were firing at noon. Still convinced of the theory, I now recall that I was *not* anxious, whence my X-fibers were probably not firing at noon. Still this conditional seems true: *If my X-fibers were firing at noon, then the theory was wrong about them.* While this one still seems true as well: *If they were firing at noon, then I was feeling anxious at noon.* In worlds where they were firing, however, the theory is right iff I was anxious, because that's what the theory says.

The third has an analogous structure to the first. *If a agrees H-wise with b, then b is H-wise thus and so* turns on whether *a* is *H-wise thus and so*; mutatis mutandis for *If a agrees H-wise with b, then a is H-wise thus and so.* In worlds where *a* and *b* do not agree *H*-wise, there is no reason that *a is H-wise thus and so* cannot be true while *b is H-wise thus and so* is false.

According to a certain book, nothing ever changes. *If the book is correct, nothing ever changes* is true, because that by hypothesis is what the book says. *If the book is correct, it says that nothing ever changes* is false; to be correct, it should say that things change, since they decidedly do. The consequents are equivalent in worlds where the book is correct.

Let me now skip ahead to the "loving" example. The choices before me are to love you or leave you; one is right and the other wrong. *I don't want to be right* is equivalent in loving-you-is-wrong worlds to *I don't want to leave you*, and *I want to love you* is equivalent in such worlds to *I prefer wrong to right. If loving you is wrong, I want to be wrong* is true, because I want to love you. *If loving you is wrong, I want to love you* is false, because as between wrong and right, I prefer right.[52]

Conditionals are away-dependent; the truth-value or correctness-value or probability of $A \rightarrow C$ is not always a function of C's behavior in A-worlds.[53] A couple of models have been suggested of how away-dependent conditionals might be accommodated. One, suited especially to absolute conditionals, involves *truthmakers*. The other, discussed in section 8, suited to local conditionals, involves *partitions*. These are not as different as they appear. The conditions on a suitable partition are akin to the conditions on truthmakers.[54] In the next section we consider the two models in turn.

XIV. Cells and Truthmakers

On the truthmaker model, $A \rightarrow C$ is true in an away-world w just in case an ($A \supset C$)-implying fact obtains there that (i) is consistent with A, and (ii) makes the most of A, that is, minimizes the extent to which $A^- \supset C$ is also implied. A fact implying $A \supset C$ the (i)-(ii) way is one thing, a fact implying $A \supset D$ in that way is another—even if C and D are true in the same A-worlds. *Nothing changes* is, for instance, *The book is correct*-equivalent to *The book says that nothing changes*. Imagine a world where things change, while the book says they don't change. That the book *says* (we're imagining) that nothing changes is a targeted truthmaker for *The book is correct* ⊃ *Nothing changes*. But *The book is correct* ⊃ *It says nothing changes* has no such truthmaker. That nothing changes would do it, but that is not a fact that obtains in the relevant world. *Correct→Stasis* is true in that world, while *Correct→Book says "Stasis"* is false. This is a clear case of away-dependence, since in worlds where the the book is correct, it says that nothing changes just if nothing indeed changes (Table 1).

Next the partition-based, or "local," route to away-dependence. To read $A \rightarrow C$ locally is to associate it, not with a single conditional probability $\pi(C|A)$, but a bunch of them $\pi(C|A\&H_i)$, H_i ranging over hypotheses about the objective set-up that affect the prevalence of C-worlds among A-worlds. These are then brought together into a weighted sum, the weights given by the prior probabilities of each hypothesis:

52 Precis of next section: Away-dependence can be arranged either with partitions or truthmakers. Truthmakers for us are sets of worlds, and partitions have sets of worlds as their cells. Some partitions are of course better than others, and some truthmakers—the targeted ones—are better than others. The good-making features of cells turn out to be not unrelated to the desiderata for a good truthmaker. Incremental conditionals can as a result be seen as *local* conditionals par excellence.

53 Gabbay (1972). Gabbay mentions unpublished work by Hans Kamp.

54 Cells and truthmakers are both conceived here as sets of possible worlds.

$$\pi(A \rightarrow C) = \pi_H(C|A) = \Sigma_i\ \pi(C|A\&H_i) \times \pi(H_i).$$

Now, why would $\pi(A \rightarrow C)$ come apart from $\pi(A \rightarrow D)$ (when C and D are A-equivalent) on this way of doing it? Our confidence in $A \rightarrow D$ is, one might think,

$$\pi(A \rightarrow D) = \pi_H(D|A) = \Sigma_i\ \pi(D|A\&H_i) \times \pi(H_i).$$

But, given that C and D are true in the same A-worlds, $\pi(D|A\&H_i) = \pi(C|A\&H_i)$, whence $\pi_H(C|A)$ is the same quantity as $\pi_H(D|A)$. It must be, then, that the *partition itself changes* when we switch to $A \rightarrow D$.

$$\pi(A \rightarrow D) = \pi_{H'}(D|A) = \Sigma_i\ \pi(D|A\&H'_i) \times \pi(H'_i).$$

So far so good—but *why* does the partition change when the consequent changes from C to D? For the same sort of reason as the targeted truthmakers for $A \supset Z$ change when Z changes from C to D. A targeted truthmaker for $A \supset C$ ($A \supset D$) has got to make the most of A, which means, in the first case, not implying C all by itself, and in the second, not implying D all by itself. C yields an unacceptable partition for $A \rightarrow C$ because it renders A irrelevant, the very reason it cannot serve as a targeted truthmaker for $A \supset C$.[55]

So—one way of getting $A \rightarrow C$ to care about C's away games is to read it incrementally: $\pi(A \rightarrow C) = \pi(C \sim A)$, for short, $\pi(\Delta)$. Another is to partition according to the factors controlling conditional probability between which you are undecided. The first approach can seen as a limiting case of the second; partitioning on Δ vs $\overline{\Delta}$ is the best one could possibly do on the score of controlling for factors affecting conditional probability. What do I mean by this? In one cell the conditional probability is π

[55] Imagine for instance that $A \rightarrow C$ is *If your mood ring is accurate, you are jealous.* A natural partition is $H_1 = D =$ *Your mood ring glows green* and $H_2 = \neg D =$ *Your mood ring does not glow green. The ring is accurate* exerts one kind of control over *You are jealous* in *Green*-worlds and another in worlds where the ring does not glow green. Shall we stick with the same partition when it comes to evaluating *If your ring is accurate it must be glowing green*? Surely not, for *Your ring is accurate* exerts no control whatever over the ring's color in either cell of $\{Green, \overline{Green}\}$. *The ring glows green* holds regardless of accuracy in the *Green*-region, and fails regardless of accuracy in the $\overline{Green}$-region. The partition we use to assess *Your ring is not mistaken* $\rightarrow$ *You are jealous* is $\{Green, \overline{Green}\}$. (Reliability makes different demands on your mood depending on which mood the ring is currently indicating.) To assess *The ring is accurate* $\rightarrow$ *It glows green*, we use $\{Jealous, \overline{Jealous}\}$. (The probability that it *says* you're jealous, if it is accurate, is the probability that you *are* jealous.) To partition in the first case on whether you are jealous, or in the second on whether the ring glows green, would be silly. One might as well try to determine the relevance of smoking to cancer by asking whether smokers with (without) cancer are likelier to have cancer than non-smokers with (without) cancer.

$(C|A\&\Delta) = 1$. In the other it's $\pi(C|A\&\overline{\Delta}) = 0$.[56] The partition Δ vs $\overline{\Delta}$ thus completely settles A's probabilistic influence on C, driving it up to 1 in the Δ (= the $C{\sim}A$) -region, and down to zero in its complement.

XV. Proper Partitioning

The absolute/local analogy is our first glimpse of how the manifold of conditionals might hang together. To take the analogy further, we would need to know how (on the local approach) the partition into different possible scenarios H_i is arrived at. This is a delicate matter for several reasons. The first and most obvious is that $A{\rightarrow}C$'s local probability is going to vary with the partition. Second, the triviality results suggest that some partitions are just bad. The local probability of $A{\rightarrow}C$ is $\pi(C)$, for instance, if the partition is C versus $\overline{C}$.

Third, A's probabilistic bearing on C may be uniformly positive across cells, or uniformly negative. This might seem to clarify matters, except that the polarities are oftentimes *reversible* by cutting cells up more finely. The logic is that of Simpson's paradox. Female applicants, the story goes, are less likely to be admitted to grad school at Berkeley than men. But they are *more* likely to be admitted to any particular department; it is just that they apply to more competitive departments. One can imagine that they are again *less* likely to be admitted, in each department, both to MA programs and to PhD programs; their advantage in departments as a whole is because they tend to apply for the less competitive degree. And so on.[57]

Fourth, local probabilities are going to be more or less objective depending on the range of acceptable partitions.[58] Incremental conditionals fare best by this standard because the one and only allowable partition is between worlds where $C{\sim}A$ is true and worlds where it is false. Otherwise the principles guiding partition choice are unclear.

How did we hit on *#1 is Holmes* vs *#1 is not Holmes* as our partition for *He accused Mrs Hudson* → *She did it*? The problems with *She did it*,

[56] $C{\sim}A$ is true, when it is, on account of a truthmaker for $A{\supset}C$. $A\&(C{\sim}A)$ therefore implies C, whence the probability of C conditional on A in the $C{\sim}A$ cell is 1. $C{\sim}A$ is false, when it is, on account of a truthmaker for $A{\supset}\neg C$. $A\&\neg(C{\sim}A)$ therefore implies $\neg C$, whence the probability of C conditional on A in the $\neg(C{\sim}A)$ cell is 0.

[57] Which statistic is relevant for any particular applicant may depend on where she is in her decision process. Stanford may be a better choice for those not decided on a department; Berkeley is a better choice if they are applying to Computer Science; the advantage goes to Stanford, maybe, if they're seeking a PhD. This is on the principle that we choose on the basis of "stable" properties of the options, properties that they retain whichever option is chosen.

[58] Douven (2008).

She didn't do it, have already been mentioned; we return to them below. What about *#1 accused Mrs Hudson* vs *#1 did not accuse Mrs Hudson*? Or *#1 said she did it*$\supset$*She did it* vs $\neg$(*#1 said Mrs Hudson did it*$\supset$*She did it*)? The reasons differ but they are reminiscent, in each case, of the constraints on targeted truthmakers: they should be consistent with A and should not waste A.

Suppose we tried to break A's bearing on C up into the difference it makes in C-worlds, and the difference it makes in $\neg C$-worlds. $\pi(C|A\&C)\times\pi(C) + \pi(C|A\&\neg C)\times\pi(\neg C) = \pi(C)$. A just drops out of the picture on this partition, because C and $\neg C$ both screen A off from the consequent. (If we divide the Berkeley applicants into those that were admitted and the rest, we will find that gender makes no statistical difference in either group.) The point is that we want C's truth-value to *vary* in each cell, lest A's contribution go unrecognized. This ought to remind us of how a targeted truthmaker for $A \rightsquigarrow C$ ($A \rightsquigarrow \neg C$) is supposed to assign A the greatest possible responsibility for the fact that A with its help entails C ($\neg C$).

Or suppose we tried to break A's bearing on C into the difference A makes in A-worlds, and the difference it makes in $\neg A$-worlds. $\pi(C|A\&A)\times\pi(A) + \pi(C|A\&\neg A)\times\pi(\neg A) = \pi(A\&C) +$ undefined. $\pi(A\&C)$ tells us nothing about the proportion of C-worlds in the A-region (for it is silent the prevalence of $\neg C$-worlds in that region). This ought to remind us, again, of how a truthmaker for $A \rightsquigarrow C$ ($A \rightsquigarrow \neg C$) is supposed to be consistent with A and to *lean* on it to the maximum extent possible. A truthmaker that repeats things already there in A (as facts implying A do) is taking for itself a job that A could have done.

So much for what we do not want; what *do* we want? I do not exactly know, but it is something like the following. We want to partition the space of worlds into scenarios that fix the "channels" over which A sends its signal to C. A partition is better, other things equal, if A's distribution in a cell is more predictive of C's in that same cell. Incremental conditionals are, again, the limiting case of this, where A's truth-value is *completely* predictive of C's both in the Δ-region and its complement: C holds in *all* A-worlds of the first kind and *none* of the second kind.

XVI. Unification (?)

But, you may say, Δ versus $\overline{\Delta}$ is not the only partition with this property. Take again the Oswald conditional: *Oswald didn't shoot Kennedy*$\rightarrow$ *Someone else did*. The obvious partition here is between *Kennedy was killed* and *Kennedy was not killed*. Partitioning on *was* vs *was not* is just like partitioning on $C\sim A$ and $\overline{C\sim A}$ as far as the math goes. *Was killed*-worlds where Oswald did not do it are all *Someone else did*-worlds, while *Was not killed*-worlds where Oswald didn't do it are all *No one else* *did*-worlds. Yet the existential fact that Kennedy was killed can hardly be considered a *truthmaker* for *Oswald didn't do it*$\supset$*Someone else did*; on the contrary Kennedy was killed by virtue of Oswald's killing him. The hypothesis of Kennedy being killed plays instead a belief-making role. Our confidence that Kennedy was killed is based on the film; it is not derived from any particular hypothesis about who killed him. Our confidence that he was killed is thus well-positioned to ground our confidence in *Oswald didn't*$\supset$*Someone else did*. The story is just like before except that truth-grounds have been replaced by epistemic grounds.

Go back to the whiteboard case and pretend this time that the only two sentences on the whiteboard are *Fish never sleep* (F) and *Goats eat tinfoil* (G). Cloris tells you F and G agree in truth-value, but this time it's not clear whether to believe her. Cloris might be a knight, who always tells the truth, or else a knave, who always lies. Partitioning on the knight and knave hypotheses is just like partitioning on $C\sim A$ and $\overline{C\sim A}$ as far as the math goes. F-worlds in the knight cell are all G, while F-worlds in the knave cell are all $\neg G$. Just as the probability of *Brad is right*$\rightarrow$*Nothing changes* comes down to the probability of Brad's claiming that nothing changes, the probability of $F\rightarrow G$ is just the probability that Cloris is a knight. Yet $F\rightarrow G$ is the opposite of an absolute conditional.[59]

The technical point is undeniable, and points us in the proper direction. The Cloris example can serve as a template for the understanding of marketplace conditionals generally. We start by listing the conditions on cells X, $\overline{X}$ that make for an absolute reading of the conditional; these are no different than the conditions on targeted truthmakers. To obtain *non*-absolute conditionals, we relax one or more of these conditions.[60] So: X vs $\overline{X}$ is a partition fit for absolute conditionals iff X and $\overline{X}$ are targeted truthmakers for $A\supset C$ and $A\supset\neg C$, respectively. Teasing the components of targeted-truthmaker-hood apart a bit to enable pointwise adjustments, X vs $\overline{X}$ is an absolute partition iff

Entailment	X entails $A\supset C$ and $\overline{X}$ entails $A\supset\neg C$	since they are *truth* makers
Explanation	X explains $A\supset C$ and $\overline{X}$ explains $A\supset\neg C$	since they are truth *makers*[61]
Consistency	X and $\overline{X}$ are consistent with A	A can't help if it contradicts X
Insufficiency	X and $\overline{X}$ are consistent with $\neg A$	A isn't helping if X implies it[62]

[59] $F\rightarrow G$ is not absolute because Cloris being a knight creates only an epistemic block to $F\&\neg G$;. It does not tell us what objectively stands in the way of the combination, and indeed presumably nothing does. This is crucial if absolute conditionals are to be aperspectival; for surplus content is unique and reasons for belief are not unique. Disagreements about $A\rightarrow C$ lose their objectivity if you are working with one sort of evidence —Cloris is a knight— and me another — e.g., Doris always wipes the board clean when she sees two truth-values represented.

[60] In some cases, tweaking others to maintain coherence.

[61] Explanation here is something like the converse of holding in virtue of.

Neededness	X and $\overline{X}$ are consistent with $\neg C$	or X wouldn't *need* help[63]
Efficiency	X ($\overline{X}$) uses all of A that it can	or X will have wasted A[64]

Weaker than **Entailment** would be a condition requiring X and A to *probabilify* C instead of entailing it, and similarly for $\overline{X}$, A, and $\neg C$. This is what happens in the Urn example. If X and A hold (you draw a red ball from Urn 1) then it is 90% likely that S (the ball will be shiny). If $\overline{X}$ and A hold (if you draw a red ball from Urn 2) then it is 90% likely that $\neg C$ (that the ball will be dull).[65]

Weaker than **Explanation** would be a condition allowing X to be, say, epistemically prior to $A \supset C$ rather than explanatorily prior. This is what we get with the Oswald example, as already discussed. Our belief that Kennedy was killed is not based on a belief regarding any particular person that *s/he* killed him. That Kennedy was killed by someone or other does not explanatorily underwrite his being killed by another person if not by this one; rather the *belief* that he was killed *epistemically* underwrites the belief that another person killed him if this one did not.

Considering **Consistency** and **Insufficiency** together, they say that X and $\overline{X}$ are logically independent of A; neither of X, A implies the other, and likewise for $\overline{X}$ and A. Weaker than this would be a condition allowing one-way implication but not two-way. A might be allowed to imply X, if C were so weak that A didn't need help; X could then be weak or trivial. X might be allowed to imply A if, although C was stronger than A, the added strength is so hard to extricate that the only truthmaker we can find for it is C. What else could serve as the truth-grounds for *Someone is sleepy*$\supset$*I am sleepy* besides the fact that I am indeed sleepy? If we want the corresponding indicative to come out true, as we presumably do, we will have to lighten up on

[62] If X (or $\overline{X}$) entailed A it would be doing itself what A might have, whence it would not likely be making the most of A.

[63] If X entailed C it would not be drawing on A at all, whence it would not likely be making the most of A.

[64] **Efficiency** is the preciser truth behind **Insufficiency** and **Neededness**. The latter hold only when the antecedent is usable, which is not always. The only *Tom is colored*-consistent truthmaker I can think of for *Tom is colored* $\supset$ *Tom is red* is the fact that Tom is red. This is not consistent either with Tom being uncolored (contra **Insufficiency**) or Tom not being red (contra **Neededness**). It is typical of determinables D and their determinates D^* that a truthmaker X for $D \supset D^*$ cannot assign any responsibility to D for the fact that X and D imply D^*.

[65] Or take the game show conditional: *If #1 accuses Mrs Hudson, she did it.* The deciders are *#1 is Holmes*, *#1 is Watson*. These are consistent with the antecedent (*#1 accuses Mrs Watson*) and make good use of it. But they do not combine with *#1 accuses Mrs Watson* to definitively resolve the consequent (*She did it*); Holmes is not always right nor Watson always wrong. The deciders here do somewhat resemble truthmakers in being explanatory, though. For *#1* to be Holmes goes some distance toward explaining why if he accuses her then she did it.

Insufficiency; the fact making $A \supset C$ true in this case, my sleepiness, is sufficient for the antecedent—for the truth of *Someone is sleepy*.

A more intriguing possibility is this. Suppose **Entailment** is weakened to allow probabilification instead of entailment. Then **Consistency** and **Insufficiency**, which together constitute a modal independence condition, might be strengthened to require probabilistic independence. Rather than requiring $\pi(A|X)$ and $\pi(X|A)$ to be less than 1, which corresponds more or less to two way non-implication, we require $\pi(A|X)$ to be $\pi(A)$ and $\pi(X|A)$ to be $\pi(X)$–which is just for the probability of $A\&X$ to be the product of the probabilities of its conjuncts. Alternatively we might require A to be probabilistically independent of the indicative conditional $A \rightarrow C$ (of which X is meant to be a close relative). This take us in the direction of Adams conditionals, since his THESIS has probabilistic independence as a consequence.

Weaker than **Neededness** would be a condition that allowed X to entail C itself, with no help from A. This opens up room for non-interference conditionals along the lines of *Whether she ate the mushroom or not, she's still very much alive.* Non-interference conditionals are intuitively speaking even more at odds with **Efficiency**, since the whole point of them is that A gets no traction on C, while the point of **Efficiency** is to demand traction. (Or at least, all the traction arrangeable. **Efficiency** is disappointed by non-interference conditionals, but it may not absolutely rule them out.)

Generalizing wildly from these examples, we might try a *focal meaning* analysis of conditionals, in which certain of them are considered paradigmatic and others are understood as departing in various ways from the standard they set. Suppose we are trying to evaluate C conditional on A relative to such and such a partition. The partition appropriate to absolute conditionals has various special properties that make it unique of its kind. Other sorts of partition fall short in one way or another. Their cells do still support $A \supset C$ or $A \supset \neg C$. There are differences though in the amount of support, and the kind, and their attitude toward the antecedent. Every step away from targeted-truth-maker-hood is a step away from objectivity and toward parochiality.

I spoke earlier of a continuum of conditionals, but that turns out to be not quite right. The fact behind the Oswald conditional (that Kennedy was killed) falls short in explanatoriness; the facts behind the Urn conditional fall short in decisiveness; the facts behind the Holmes conditional (*#1* is probably Holmes) are not decisive but do seem somewhat explanatory. The facts behind a non-interference conditional are decisive and explanatory, but not targeted, since they require no help from the antecedent. One should speak really of a spider web of conditionals, with surplus content conditionals at the center.

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Formal Details

Here is one way of implementing the proposal formally. The first two steps recapitulate van Fraassen's truthmaker semantics in "Facts and Tautological Entailment."

Models A *model* $\mathcal{M}$ of propositional language $\mathcal{L}$ is a triple $< W,F,V>$ where

1. W is a set of worlds
2. F is a collection of privileged subsets of W (the "facts"), closed under certain operations
3. V is a function assigning to each atomic sentence a a fact $\mathtt{a}$

Truthmakers τ is a *truthmaker* (*falsemaker*) for φ iff

1. S is an atomic sentence a, and $\tau = \{\mathtt{a}\}$ $(\tau = \{\overline{\mathtt{a}}\})$[66]
2. S is $\neg P$, and τ is a falsemaker for P (τ is a truthmaker for P)
3. S is $P \vee Q$, and τ makes P true or Q true (τ is the union of falsemakers for P and Q)
4. S is $P \& Q$, and τ is the union of truthmakers for P and Q (τ makes P false or Q false)

S tautologically entails P, van Fraassen shows, if every truthmaker for S contains a truthmaker for P. Requiring further that every truthmaker for P is contained in one for S gets us (what in *Aboutness* I called) *inclusive* entailment.[67]

Now we expand $\mathcal{L}$ in the usual way to a language $\mathcal{L}^+$ any two of whose sentences, even those with conditionals in them, can be combined into a further conditional $P \rightsquigarrow Q$. A set of truthmakers and falsemakers must be assigned to each of the new sentences. A truthmaker τ for $P \supset Q$ is *wasteful* in w iff (it obtains there and) a σ obtains

66 $\overline{\mathtt{a}}$ is the set of worlds not in $\mathtt{a}$.
67 Compare Fine (2013)

in w that uses strictly more of P in this sense: for some P^- implied by P, σ does not nontrivially imply $P^- \supset Q$ and τ does; but not vice versa. A truthmaker is efficient iff it is not necessarily wasteful.

Assume that P and Q have already been endowed with truthmakers and falsemakers.

DIFFERENCE-MAKERS A difference-maker for P and Q ($\neg Q$) is a nontrivial, efficient truthmaker for $P \supset Q$ ($P \supset \neg Q$).

CONDITIONALS $P \rightsquigarrow Q$ is true (false) in w if a difference-maker for P and Q (P and $\neg Q$) obtains there.[68]

Examples

1. $p \rightsquigarrow p\&q$................ is true in the same worlds as q
   Goats eat cans $\rightsquigarrow$ *They eat cans and bottles* $\Leftrightarrow$ *Goats eat bottles*
2. $p \supset q \rightsquigarrow p \equiv q$ ....is true in the same worlds as $q \supset p$
   Goats eat cans if bottles $\rightsquigarrow$ *They eat cans iff they eat bottles* $\Leftrightarrow$ *Goats eat bottles if they eat cans.*
3. $p \vee q \rightsquigarrow p$ .... is true in the same worlds as $\neg q \vee p$
   Goats eat cans or bottles $\rightsquigarrow$ *Goats eat cans* $\Leftrightarrow$ *Goats either do not eat bottles or do eat cans*
4. $p \vee q \rightsquigarrow p \dot{\vee} q$[69] .... is true in the same worlds as $\neg(p\&q)$
   Goats eat cans or bottles $\rightsquigarrow$ *They eat ONE of cans and bottles* $\Leftrightarrow$ *They don't eat both cans and bottles*
5. $p \vee r \rightsquigarrow (pq \vee r)$ ....is true in the same worlds as $q \vee r$
   Goats eat cans or bottles $\rightsquigarrow$ *They eat cans and tinfoil, or else bottles* $\Leftrightarrow$ *They eat tinfoil or bottles*
6. $p \equiv q \rightsquigarrow p \equiv r$ ....agrees in all worlds with $q \equiv r$
   Goats eat cans iff they eat bottles $\rightsquigarrow$ *They eat cans iff tinfoil* $\Leftrightarrow$ *They eat bottles iff they eat tinfoil*

68 This allows $P \rightsquigarrow Q$ and $P \rightsquigarrow \neg Q$ to hold together. We could alternatively count these cases gappy.

69 $\dot{\vee}$ is exclusive disjunction