# *A reply to new Zeno*

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A new Zeno paradox has been devised that seems rather more challenging than the old ones. It begins like so:

> A man decides to walk one mile from A to B. A god waits in readiness to throw up a wall blocking the man's further advance when the man has travelled 1/2 mile. A second god (unknown to the first) waits in readiness to throw up a wall of his own blocking the man's further advance when the man has travelled 1/4 mile. A third god … &c. *ad infinitum*. It is clear that this infinite sequence of mere *intentions* (assuming the contrary to fact conditional that each god would succeed in executing his intentions if given the opportunity) logically entails … that the man will be arrested at point A; he will not be able to pass beyond it, even though not a single wall will in fact be thrown down in his path. (Benardete 1964: 259–60)

So far this is just a surprising result. But a contradiction ensues if we stipulate – what seems anyway implicit in the original set-up – that the man does not stop unless a barrier is put in his way. Now we can infer from the fact that no walls are thrown down that he eventually makes it to B, and so in particular makes it past A. So he both does and doesn't proceed beyond A.

Borrowing from Priest 1999, we can see the paradox as disclosing an unexpected inconsistency between some innocent-looking assumptions. Suppose that object $\alpha$ is gliding in from the left along an infinite number line, with $A$ and $B$ at the $x = 0$ and $x = 1$ marks respectively. The gods are waiting to set up their walls if $\alpha$ goes anywhere to the right of $A$, that is, if $\alpha$'s position on the $x$-axis becomes positive. Let '$Rx$' mean that $\alpha$ reaches point $x$, and '$Bx$' mean that a barrier is put up at $x$ while $\alpha$ is to the left of $x$, that is, while $\alpha$ is at some $y < x$. Our first three premises describe what Priest calls 'general features of $\alpha$'s motion'. First, $\alpha$ passes through all points to the left of points it passes through:

(1) $(Rx \ \& \ y < x) \rightarrow Ry$

Second, $\alpha$ can't pass through barriers:

(2) $(By \ \& \ y < x) \rightarrow {\sim}Rx$

Third, barriers are the only thing that can prevent $\alpha$'s forward advance:

(3) $(\forall x\,(x < y \rightarrow {\sim}Bx)) \rightarrow Ry$

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One final assumption gives the rules for barrier-introduction. A barrier is put up at $x$ if $x = 1/2^n$ for some $n$, and α makes it halfway to $x$, that is to the $1/2^{n+1}$ mark:

(4) $$Bx \leftrightarrow (\exists n(x = 1/2^n) \,\&\, R(x/2)).$$

From (1), (2), and (4), we see that α will advance no further than $x = 0$. For suppose that α makes it to $x = 1/2^n$. By (1), α has already advanced to $x/4$, whence (4) tells us that a barrier has been put up at $x/2$. But then α never makes it to $x$, by (2). α can't get past 0 without reaching $1/2^n$ for some $n$, so α is stopped at 0. That being so, though, we can conclude by (4) that no barriers are ever put in α's path. And now (3) tells us that α gets as far to the right as you like, contrary to our earlier conclusion that it stops at $x = 0$.

What are we to make of this? Priest says, plausibly enough, that '(1) to (3) are features of continuous motion, and are true in this world'. Although (4) is not true in this world – 'there are no parts of the world where passing certain spots brings barriers spontaneously into existence' – it certainly seems as though it *could* be. 'There could,' for instance,

> be a demon. The demon is bound by the laws of physics, and so, in particular cannot suspend the laws of motion (1) to (3). Yet it might 'mine' an area of space in accordance with (4).... Alternatively, to use Benardete's example, there might be an infinitude of such demons, each of which has the appropriate intention. (Priest 1999: 2)

There are imaginable circumstances, then, Priest reasons, in which 'motion produces contradictions'. And this seems little different from saying that motion itself 'could be contradictory'. Are these conclusions warranted?

They are if we can take it for granted, as Priest does, that if there were 'an infinitude of... demons, each of which has the appropriate intention,' then (4) would be true, that is, the system of barriers would work as advertised. But what if the demons aren't able to do as they intend? Benardete is more careful. He sees the need to assume as well that each demon 'would succeed in executing his intentions if given the opportunity'.

A case can be made, though, that the stronger assumption is still not strong enough. For it isn't in general true that 'if each member of a group is able to do as she intends, whatever the others do, then the members can execute their intentions en masse'. Imagine that our infinite series of demons is asked to call off YES's and NO's in reverse order, with demon $n$ calling off after demon $n + 1$. (They can do this in finite time if the $n$th demon calls off at $t = 1/2^n$.) Each demon wants to be the first to say YES, but would rather say NO if she cannot be first. Each demon thus forms the intention of calling YES if and only if all the earlier-calling demons have called NO, and calling NO if and only if some earlier-calling demon has called YES.

If we focus on any particular demon, there is nothing to stop her from executing her intention, given the opportunity. All she has to do is call YES if her predecessors have called NO, otherwise NO. Does it follow that there is nothing to stop the demons from fulfilling their intentions as a group? Logic stops them. Either some demon says YES or none do. If none do, then all should, because their predecessors have all said NO. If some do, then for some $n$ the $n$th demon says YES. If she has been faithful to her intentions, then for all $m > n$, the $m$th demon has said NO. But then the $(n + 1)$st demon's predecessors have all said NO, and so the $(n + 1)$st demon should have called not NO but YES. Not all of the demons, then, can have stuck to the plan.

Now let's go back to the original paradox, except this time let's forget about $\alpha$. The demons have decided, quite apart from any desire to thwart $\alpha$, to do some reverse-order wall-building. Each demon$_n$ resolves that

(R) I will put up my wall if and only if no demon$_m$ with $m > n$ has put up hers.

It is clear from the argument given that not all the demons can act in accordance with (R). At least one will put up a wall despite walls having already been put up, or refrain from doing so despite no walls having yet been put up.

But then it should be equally clear that *if $\alpha$ were to make it past 0*, not all of the demons would be able to carry out their original resolution

($R_0$) I will put up my wall if and only if $\alpha$ makes it half-way to me.

Should $\alpha$ make it past 0, then given the stated constraints on its motion – in particular the fact that $\alpha$ gets to $1/2^{n+1}$ iff no walls have yet been put in its path – success in carrying out ($R_0$) would ipso facto be success in carrying out (R).

Where does this leave us? The paradox turns on a lemma: $\alpha$ stops at 0. It has to stop at 0, for we get a contradiction when we try to work out $\alpha$'s progress through the gauntlet of demons. But maybe the reason for the contradiction is that the gauntlet is incoherent in itself. Or rather, it is saved from incoherence only by the assumption that $\alpha$ stops at 0.

Installing a desired conclusion as the only escape route from what would otherwise be a contradiction is a time-honoured method of proof. (The usual example is 'either this statement is false or God exists'.) The new Zeno paradox might be seen as fitting the pattern as well:

either $\alpha$ stops at 0 or something impossible happens: the demons act in accordance with (R).

But even this gives the paradox too much credit, since there is another way out. All we can conclude on the basis of the given conditions is that

> either $\alpha$ stops at 0, or the demons act in accordance with (R) *if they act in accordance with* ($R_0$).

Either $\alpha$ stops at 0, or it continues past 0 and the demons are forced to face up to their inability to carry through on their promises.

The *general* point is that $\alpha$'s continuing past 0 puts the security system to a test that it cannot possibly pass. Let it be that the demons 'mine' the $x > 0$ region so that a wall comes down automatically if $\alpha$ advances too far. If $\alpha$ advances too far, the mines will not work as advertised – not because they are defective but because logic doesn't allow it. If there's a paradox here, it lies in the difficulty of combining individually operational subsystems into an operational system. But is this any more puzzling than the fact that although I can pick a number larger than whatever number you pick, and vice versa, we can't be combined into a system producing two numbers each larger than the other?

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