RELEVANCE WITHOUT MINIMALITY[1]

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1. INTRODUCTION[2]

A notion that comes up everywhere in philosophy is that of a circumstance "contributing" to a result or outcome—or being a "factor" in, or "helpful" or "relevant" to, the result or outcome. One is looking in most cases for a *Q* that is *wholly* helpful: free of irrelevant accretions making no real difference.

Causes should bear positively on their effects. Material to which an effect is not beholden should be kept as far as possible out of its cause. An argument's premises, or the assumptions

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employed in a proof, should help to make the case for its conclusion. If a premise can be dropped without invalidating the argument, it probably shouldn't have been there in the first place. Grounds should contribute to what they ground, both in toto and throughout. That it would redress an injustice is a reason for φ-ing only if its redressing the injustice counts in favor of φ-ing. Insofar as other properties of φ-ing (it is normally done at night) do not count in its favor, these other properties do not form part of the reason for φ-ing. An observation does not confirm a hypothesis if it is irrelevant to whether the hypothesis is true; one would not expect it to figure in the evidence for that hypothesis.[3]

This last example, of confirming *P,* or figuring in the evidence for it, helps to clarify the kind of relevance at issue. Hempel distinguishes three progressively more elaborate types of confirmation—absolute, comparative, and quantitative—in order to focus attention on the first (Hempel[1945]). Quantitative confirmation theory tries to develop measures of the extent to which *Q* confirms *P*. Comparative confirmation theory tries to make sense of *Q* confirming *P* *more* than *Q'* confirms *P*. Absolute confirmation is a binary affair, both in involving only two elements—*Q* and *P*— and allowing only two verdicts—*Q* confirms *P*, or else it fails to confirm *P*. Hempel mentions comparative and quantitative confirmation only to put them aside for a later stage of the investigation.

Relevance in the sense of this paper is a binary affair too. *Z* contributes to *Y*, or it does not, period.[4] Not a lot will be said about comparative helpfulness, or degrees of helpfulness. Various other subtleties will be set aside as well. Our focus will be on actual, rather than generic, or potential helpfulness. This means, first, that *Z* is helpful to *Y* only if both obtain.[5] It means too that a factor that normally works against *Y*—*Y* holds, if it does, *despite* this factor—may yet be

[3] I am hedging a bit because *Q* need not be "intrinsically" relevant to be relevant in the circumstances.

[4] Helpfulness may be contingent on other facts, but they are not among its relata.

[5] Much as both need to obtain for *Y* to hold "despite" *Z*. Helpfulness in our sense is roughly the opposite of despiteness.

helpful to it on a particular occasion, and vice versa. Likewise a normally neutral *Z* may join forces with *Y*'s friends on some occasions, and its enemies on others.

Plan of the paper: Relevance is usually explained in terms of notions like *minimality*, *difference-making*, *essentiality*, and *non-redundancy*. The standard explanation is reviewed in the next two sections, first from an analytic, and then a quasi-historical perspective. We will see that it does not get to the heart of things: *Z* can still contribute to *Y* even if *Z* does not figure indispensably in the conditions for *Y,* and even where minimality considerations do not apply. The problem is seen to have hyperintensional aspects. A hyperintensional diagnosis is suggested and a possible solution sketched in terms of "focussed" minimality, or minimality where a certain subject matter is concerned. We sum up at the end and lay out further directions for research.

2. DEPENDENCE

Asked what it means for *Z* to contribute to *Y* or be a factor in its occurring, our first thought is that *Y* should counterfactually depend on *Z*, that is, *Y* would not have obtained if not for *Z*. Writing $\gg$ for the counterfactual conditional and using an upper bar for negation,

[*C*1] *Z* contributes to *Y* just if: $\bar{Z} \gg \bar{Y}$

This will not long satisfy us, though, for a couple of reasons. One is that *Z* and *Y* will in some applications (grounding, entailment,....) be necessary. $\bar{Z} \gg \bar{Y}$ will in that case be a counterpossible conditional. Counterpossible conditionals are at least as theoretically elusive as positive relevance, and raise some of the same problems, e.g., both are hyperintensional.

The second reason not to rest too much on counterfactuals comes from the theory of

causation. Z can contribute causally to Y even if Y would still have obtained (on some alternative basis) in Z's absence. The mismatch is often explained as follows.[6] Y depends on Z if an X obtains with four properties:

(i) X contains Z;

(ii) X suffices for Y ($X \Rightarrow Y$);

(iii) $X \backslash Z$ does not suffice for Y ($(X \backslash Z) \not\Rightarrow Y$); and

(iv) Y is not overdetermined—there is no backup condition B (actual or counterfactual) that would do the job in X's absence.

But, the story continues, granted that Y does not *depend* on Z if Y is overdetermined, why should this undermine Z's claim to be making a contribution? Whether there are, or would be, other contributors about, even ones sufficient for Y, seems just irrelevant to the issue of whether Z itself contributes to Y. This suggests our focus should not have been on dependence, but a deeper fact (defined by (i) - (iii)) that *underlies* dependence:

[$C2$] Z contributes to Y just if: Z is part of an actual X such that $X \Rightarrow Y$, but $(X \backslash Z) \not\Rightarrow Y$.

Merely counterfactual backups drop out of the picture on this approach. Alternative actual backups are taken in stride and seen as posing no threat. X need not be in any sense unique, on

[6] Kment[2014], Strevens[2007].

[$C2$], for Z to qualify as helpful to Y by figuring essentially in X. Z achieves relevance by pulling an "almost" sufficient condition A (aka $X \backslash Z$) over the finish line: A does not itself suffice for Y, but $A+Z$ suffices.

A problem remains, however. Z could pull A over the finish line even if it was partly irrelevant to Y, provided it was also partly relevant. Did Socrates die because he drank hemlock in a toga? Of course not. But drinking-hemlock-in-a-toga is just as helpful to his death as drinking hemlock, to go by [$C2$]. If adding hemlock-drinking to the right sort of insufficient condition A yields a sufficient condition for death, then adding hemlock-drinking-in-a-toga does too.

Here we can just double down on the idea behind [$C2$]. Rather than requiring only of Z that it be essential to X qua basis for Y, we should ask *everything* in X to be essential to it qua basis for Y.

[$C3$] Z contributes to Y just if: an X obtains such that $Z \leq X$, $X \Rightarrow Y$, and $\forall U < X\ (U \not\Rightarrow Y)$.

The new requirement reaches down to Z's parts, since these will also be part of X by transitivity of part/whole. Given that an X including the toga can be cut down to a no less sufficient U leaving the toga out, drinking hemlock in a toga does not count by the new rule as a factor in Socrates' death.

Another way to formulate the rule is that Z contributes if it is contained in a *minimal* sufficient condition for Y, a sufficient condition X whose proper subconditions are always insufficient. This, the minimal sufficiency model of relevance, is what we are going to criticize in this paper. It admits, like any philosophical model, of various refinements.[7] But we will not bother too much about these, since they do not affect the problem we're coming to. That problem runs deep, and is not easily tweaked away.

The problem, formally speaking, is that not everything *has* a minimal basis. We do not want to conclude from the fact that sufficient conditions for Y always contain smaller such conditions that none of these are wholly, pervasively helpful to Y. Especially if nothing counts as helpful at all except by participating in a sufficient X that is helpful through and through.

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[7] For instance one might want to add, for certain purposes, a ``contiguous chain'' requirement along the lines of Kim (1973).

The problem intuitively speaking is that *X,* to be wholly helpful, need only be wholly *welcome* from *Y*'s perspective. So far is this from requiring *X*'s small parts to be one and all essential to *X*, qua sufficient condition for *Y*, they can be one and all *inessential*. *X* can be composed of elements that would none of them be missed, though of course large enough combinations of them would be missed.

The intuitive problem comes out more clearly if we consider the idea of "extra help." This (utterly banal) idea is a contradiction in terms, on the minimal sufficiency model. To see why, suppose *Z* was extra, that is, beyond what was strictly needed. Then the remainder sufficed; so, *Z* plus that remainder was not minimally sufficient; so *Z* was not helpful after all. This runs completely counter to intuition. Say the winning team in a tug of war is larger than necessary. This hardly means that some strange magic occurred, in which a team achieved victory with no help from its members. That is what we have to say, though, if particular rope-tuggers, to be helpful, must pull an otherwise losing team over the line into winner-land.

3. HISTORY

I want now to re-approach the question "historically" (note the scare quotes). When did minimality pressures first begin to make themselves felt? When do we first encounter the problem just noted—that a development *Z*, to be welcome from *Y*'s perspective, needn't be a "sine qua non" of *Y*, even in the sense of being a that-without-which-*X*-would-not-suffice?

Sufficiency had a long run in philosophy before anyone got worked up about irrelevant add-ons. There was the Principle of Sufficient Reason. Causes were events given which the effect was sure to follow. Classical validity was a matter of premise-truth sufficing for the truth of the conclusion. Grounds for a higher-level fact were, and sometimes still are, items or conditions prior to that fact and sufficient for it.

An obvious worry about these proposals is that they put a lower bound on, say, the cause, but not an upper bound, since $X \Rightarrow Y$ is monotonic in X (X^+ suffices if X does). Causation and the like are more discerning. Socrates died not because he drank the hemlock in a toga, but because he drank the hemlock. The existence of even primes is grounded in 2 being an even prime, not 2 being an even prime while 9 is an odd non-prime.

If sufficiency allows causes to get too big, we might think of asking them also to be *necessary* for *Y*. Hume considers this in the *Treatise*, but rejects it, on the basis that effects need not have been caused at all, let alone by their actual causes.

> If we define a 'cause' to be *An object precedent and contiguous to another, and where all the objects resembling the former are similarly precedent and contiguous to objects that resemble the latter*, we can easily grasp that there is no absolute or metaphysical necessity that every beginning of existence should be preceded by such an object (Hume[1740 /2003], Bk I, section 14, "Of the Idea of Necessary Connexion.")

The issue for us concerns *natural* necessity, rather than metaphysical, and *Y*'s specific cause rather than its being caused at all. But specific causes are not naturally necessary either, for Hume. From *objects resembling X are always succeeded by objects resembling Y,* it does not follow that *Objects resembling Y are always preceded by objects resembling X.*[8]

Hume does appreciate, even in the *Treatise*, that causes as he officially defines them are liable to be overloaded with extraneous detail. For we find him in the very next section (*I*, *15*, "Rules by which to judge of causes and effects") looking for ways to block this:

[8] Similarly a truth does not have only one possible truthmaker, and there is more than one possible reason for doing a thing

> where several different objects produce the same effect, it must be by means of some quality, which we discover to be common amongst them...in order to arrive at the decisive point, we must carefully separate whatever is superfluous, and enquire by new experiments, if every particular circumstance of the first experiment was essential to it.

Hume suggests here a different way of keeping *X* within bounds. Rather than requiring causes to be necessary—so that *Y* no longer *holds* given just part of *X*—he asks them only to be non-redundant—*Y* is not *ensured* by just part of *X*. This becomes in the *Enquiry* (Hume[1740 /2006]) a full-blown proportionality requirement:

> we must proportion the [cause] to the [effect] and can never be allowed to ascribe to the cause any qualities, but what are exactly sufficient to produce the effect.[9]

A proportional cause is an *X* such that *X* suffices for *Y and nothing less suffices*.[10]

[*P*] *X* is proportional to *Y* ($X \propto Y$) iff

[9] "A body of ten ounces raised in any scale may serve as a proof, that the counterbalancing weight exceeds ten ounces; but can never afford a reason that it exceeds a hundred" (Hume[1740 /2003]).

[10] "Proportionality" in this paper is analogous, but not identical, to the notion at work in Yablo [1992b] and Yablo[1992a].

(i) X suffices for Y ($X \Rightarrow Y$)

(ii) for all $X' \leq X$, if $X' \Rightarrow Y$, then $X' = X$

Of course, we are often interested in "contributory" causes that are not sufficient, and hence not proportional. But Hume has an easy way to bring these on board. Z contributes to Y if it is *contained* in some proportional cause X of Y:

[*H*] Z contributes to Y ($Z \rightsquigarrow Y$) iff an X obtains such that $Z \leq X$ and $X \propto Y$.

By the *Humean Package* (*HP*), we'll mean these two ideas together. The first idea: X is proportional to Y just if it is minimally sufficient for Y. The second: Z contributes to, or is relevant or helpful to, Y just if Z is contained in a proportional X. What the two together offer is an account of relevance in terms of the prima facie much clearer notions of sufficiency and minimality.

The Humean Package has a lot going for it. It is powerful and illuminating and deals correctly with a great many cases. And it is adaptable. [*P*] and [*H*], since they do not contain the word "cause," offer a general template that is potentially of very wide application. Indeed it is hard to think of an area of philosophical inquiry that hasn't employed the template. The Hypothetico-Deductive model of confirmation is Humean in spirit; E confirms H just if H figures essentially in some suitable E-entailing body of information. An action's right-making features, on one account, are those included in some condition that is minimally sufficient for its rightness. Current theories of presupposition projection emphasize the "relevance," explained in difference-maker terms, of an embedded sentence's truth-value to the truth-value of the whole.[11] A recent

[11] Schlenker[2008]. A close cousin of the minimality problem is raised in Schlenker[2009], p. 52-3.

paper defines "*Q* is a difference-making ground for *P*" like this:

> for some scenario *S* which contains a full ground of *P*, *S* minus the fact that *Q* does not contain a full ground of *P* (Kra¨mer and Roski[2017], with inessential relettering).

How can the fact *Q* that 5 is prime be relevant to the fact *P* that there are primes, when that fact is assured independently? A scenario *S* consisting precisely of 5 and its indivisibility by 2, 3, and 4 contains thereby a full ground of *P*. No lesser scenario contains a full ground, and in particular *S* minus the fact *Q* of 5's indivisibility by 2 does not fully ground *P*. 5's oddness contributes to the existence of primes because it makes the difference between a minimal ground for primes' existence and a near-ground.

4. EXTRA HELP

The problem, again, is that a *non*-minimal condition *X*—one with elements that it doesn't need, to suffice for *Y*—can still be wholly, entirely helpful. Extra help is still help, and, what is more, sometimes it is the only kind of help around.

One example of this sort comes from Zeno. A solid sphere takes up space. It has measure 1, say. The sphere's component points are helpful, surely? Certainly they are helpful en masse—en masse they just *are* the sphere. And it is hard to see how they could be helpful together, if they are irrelevant individually. The fact remains, though, that since each point has measure 0, they

would none of them be missed. None of the sphere's component points lies in a minimal subregion of the same measure, simply because there *are* no minimal subregions of the same measure.[12]

Hume would presumably have known, when he wrote the *Enquiry*, of Zeno's paradox of measure. He would not have known of the example to come, for it goes back to events taking place the year (1748) the *Enquiry* appeared. God is pleased, let us say, if and only if he is praised infinitely many days. Being praised *every* day should be pleasing, surely.[13] But no, not if we go by the Humean Package. The reason was noted in effect by John Newton in *Amazing*

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[12] Skyrms[1983] is an interesting discussion. "Zeno's paradox of measure rests on the following premises:

(I) Partition: [the sphere] can be partitioned into an infinite number of parts such that

(II) Measurability: the concept of magnitude applies to the parts.

(III) Invariance: the parts all have equal positive magnitude, or zero magnitude.

(IV) Archimedean Axiom: there are no infinitesimal magnitudes.

(V) Ultra-Additivity: the magnitude of the whole is the sum of the magnitudes of the parts.

Ancient attempts to answer Zeno focused largely on (I) and (II). Doctrines of finite indivisible magnitudes (certainly Epicurus and probably Democritus and Leucippus) rejected (I). Aristotle rejected (I) and (II). It is possible that a doctrine of infinitesimal indivisible magnitudes was also current (possibly held by Xenocrates, possibly by Democritus) which rejected (IV). (III) could have also been challenged by a holder of a doctrine of infinitesimal magnitudes. (V), Ultra-Additivity appears to have been accepted without question by every party to the dispute. It is ironic that it is just here that the standard modern theory of measure finds the fallacy" (235).

[13] We assume the future is infinite.

Grace (1779):[14]

> When we've been here ten thousand years
> Bright shining like the sun
> We've no less days to sing His praise
> Than when we'd just begun.

Singing *every* day is out of proportion with the effect, on Hume's definition, since God would still be pleased if we waited 10000 years before beginning. And of course the same is true for any other set of days one might choose. There *is* no least infinite set of days. Every praise-day is helpful to the cause, but not because it figures in a minimal sufficer.

The moral is clear. Minimality had better not be required for relevance, because you can't always get it. As if that weren't enough, minimality is not required even when you *can* get it. The pope's crown was once supposedly made of three smaller crowns. Suleiman the Magnificent, not to be outdone, had *four* crowns in his crown. Suleiman's crown was wholly relevant to *There are crowns*. But you could lop the upper sub-crowns off and still have a sufficient condition for the sentence's truth. Here we *can* point to a minimal sufficient basis for *There are crowns*. But there is no reason to do so. Suleiman's total crown is no less helpful for being four times larger than necessary.

The US Senate cannot conduct certain kinds of business unless 51 members are present (a quorum). Let us say the Senate is not "in order" without a quorum. Suppose that 52 senators are present on a given occasion. They all arrived at the same time and the situation is in other ways symmetrical. The presence of these senators—the Gang of 52, let's call them —seems wholly

[14] Newton's religious phase began in 1748, when his ship nearly went down in a storm off the coast of Ireland. He was returning from Africa, where he had been first a slave trader, then himself enslaved to a colleague's African wife. The ship miraculously righted itself and Newton promised to change his ways. He was ordained as an Anglican priest in 1764. *Amazing Grace* was written a few years later.

helpful to order obtaining. True, there is a Gang of 51 present as well which also suffices, in fact there are 52 such gangs. Somehow though this does not detract from our initial judgment. The Gang of 52 is wholly relevant despite the fact that not all its members had to be there.

5. PREVIOUS PROPOSALS

That *X* can still be wholly relevant to *Y*, even if not all of it is needed, has not gone entirely unnoticed. Fine makes the point in connection with truthmaking (proportional truthmakers in his parlance are *exact* rather than *inexact*) (Fine[2017]).[15] Humeans in effect

> take the exact verifiers to be the minimal inexact verifiers, those that inexactly verify without properly containing an inexact verifier.

But while piling on random extra facts may produce an inexact verifier,[16] piling on additional exact verifiers seems different.

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[15] "With inexact verification, the state should be at least partially relevant to the statement; and with exact verification, it should be wholly relevant. Thus the presence of rain will be an exact verifier for the statement 'it is rainy'; the presence of wind and rain will be an inexact verifier for the statement 'it is rainy', though not an exact verifier" (Fine[2017])

[16] "On our understanding of verification as relevant verification, it should *not* be supposed that if *f* verifies a truth *A* then *any* 'larger' fact *f•g* must also verify *A*" (Fine[2012b], emphasis added; see also Fine[2012a]).

> Given the facts $f, g, h, \ldots$, we take there to be a *composite* fact or *fusion* $f \bullet g \bullet h \bullet \ldots$ that is the 'factual conjunction' of the component facts $f, g, h, \ldots$, obtaining just in case all of the components facts obtain; and we shall suppose that whenever the facts $f, g, h, \ldots$ verify the truth A, their fusion $f \bullet g \bullet h \bullet \ldots$ also verifies A...

The presence $r \bullet w$ of rain and wind exactly verifies *It is rainy or windy*, one might think, without minimally verifying it. The presence of both may be more than $R \vee W$ needs, but there is nothing in $r \bullet w$ that is irrelevant to the statement's truth.

Kratzer's theory of exemplification strikes a similar note. The fact of two teapots exemplifies *There are teapots,* she says, despite its non-minimality. But the fact of a teapot and a dog does not. Why is the extra dog more of a problem than the extra teapot? Kratzer has an interesting idea about this: the parts of a P-exemplifying situation s must "earn their keep" by figuring crucially, not perhaps in s itself, but in a minimal P-verifying part of s.

> s exemplifies P iff for all s' such that $s' \leq s$ and P is not true in s', there is an s'' such that $s' \leq s'' \leq s$, and s' is a minimal situation in which P is true. (A minimal situation in which P is true is a situation that has no proper parts in which P is true.) (Kratzer[2002]: 660)

The fact of two teapots exemplifies *There are teapots* (P) despite its non-minimality because everything in it is part of some minimal P-verifier or other. Here s and P are like our X and Y and exemplification is like being-sufficient-for-and-wholly-helpful-to.

Kratzer's theory does loosen the bonds between relevance and minimality. But

minimality is still playing its same old role one level down; a non-minimal verifier still needs to contain minimal verifiers. It is a problem, then, if "a statement may have inexact verifiers without having any minimal verifiers" (Fine[2017]).[17]

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[17] Kratzer is perfectly well aware of this. Her example is *There are infinitely many stars* (numbered (7) in her paper).

> If the proposition expressed by (7) is the proposition *P* that is true in any possible situation in which there are infinitely many stars, we are in trouble. [The] definition would predict that there couldn't be a fact that makes *P* true, ... Situations with five or six stars, for example, ... are not part of any minimal situation in which *P* istrue (ibid., 662)

Her response, intriguingly from our perspective, appeals to subject matter. She notes that there is a reading of (7)

> that the German sentence (8) brings out more clearly.
>
> (8) Sterne gibt es unendlich viele.
>
> Stars are there infinitely many.
>
> As for stars, there are infinitely many of them.
>
> In (8), the common noun "Stern" has been topicalized. The proposition expressed by (8) might now be taken to be the proposition *Q* that is true in a situation *s* iff (i) *s* contains all the stars in the world of *s*, and (ii) there are infinitely many stars in *s*. Consequently, if *Q* is true in a world at all, there is always a minimal situation in which it is true, hence there is always a fact that exemplifies it (Ibid., 662).

But, although (7) *can* be understood so that it comes out with minimal verifiers, this is not the only way of understanding it. And there might be other examples where topicalization is not an option.

Similarly a cause might still be wholly helpful to an effect, even if all its sufficient parts contain smaller such parts all the way down. Imagine a detector that buzzes when presented with a line segment of any positive length.[18]

Infinitary relevance can *sometimes* be dealt with as follows.[19] Consider again Zeno's Paradox of Measure. How do the individual points in a sphere contribute to its volume, when each point is of measure zero? Well, the points are *collectively* relevant, and none is more relevant than any other. Perhaps Y is wholly helpful to X if

(1) Y subdivides into the Y_i's

(2) X fails if *all* the Y_i's fail

(3) one Y_i is as relevant to X as another

Or, looking back at Amazing Grace, we might reason as follows. The *number* of praise- days does not shrink if we add one more day, but the *set* does shrink. And cardinality considered as a measure on sets is a coarsening of membership; size in the how-many sense is monotonically grounded in size in the membership sense. Perhaps Y is wholly helpful to X if

(1) X is to the effect that Y is at least so big by a certain measure

(2) that measure is monotonically grounded in another, finer measure

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[18] Yablo[2017a].

[19] These ideas were prompted by an observation of Williamson's about content-parts in propositional logic. B is analytically contained in A, I had proposed, if $\forall\alpha\ \exists\beta\ \beta\subseteq\alpha$ and $\forall\beta\ \exists\alpha\ \beta\subseteq\alpha$; the Greek letters range over minimal models of A and B. Williamson pointed out (p.c., 2006) that containment continues to make sense in infinitary settings where A's models are all non-minimal. *Infinitely many atomic truths exist* appears, for instance, to contain *Atomic truths exist*. (For analytic containment see Angell[1989], Correia[2004], Yablo[2014]:59, and Fine[2015a]).

(3) each Y_i bears on Y's size by this finer measure

Both ideas are interesting and potentially useful. But I don't want to pursue them here, for two reasons. The first is that they seem insufficiently general. (Why should X be to the effect that Y is "at least so big by a certain measure"?) The second reason is that they miss a crucial aspect of the problem, as we are about to see.\

6. HYPERINTENSIONALITY

One problem for the Humean package ([*H*] and [*P*]) is that minimality is not always available. A second is that minimality is not always even desirable. Now we turn to a third issue, which is different but not unrelated.

Humean proportionality is "intensional": if X and Y are necessarily equivalent to X^* and Y^*, then Y is proportional to X only if Y^* is proportional to X^*. For it looks as if proportionality is defined by [*P*] in terms of sufficiency and minimality, which are themselves intensional.[20]

Is the relation of "being entirely relevant" intensional? It is not. An example on the X side: *His praise is sung infinitely many days* (X) is true in the same worlds as *His praise is sung infinitely many days after 12019* (X^*). Singing every day starting now (Y) is wholly helpful to X, but overkill when it comes to X^*. Singing today is absolutely beside the point when it comes to singing infinitely often in the distant future.

[20] I say "looks as if" because the relation $\leq$ plays a role as well. It could turn out that necessary equivalents are not freely substitutable on the right hand side of $U \leq Y$. See section 7.

An example on the *Y* side:[21] In Alternative Eden, there are infinitely many apples on the Tree of Life but only one, *BadApple*, on the Tree of Knowledge of Good and Evil. Eve can't recall God's precise instructions, and decides to check it out with the serpent:

> Eve. What did God allow me to do again?
>
> Serpent. Hmmmm, I am not sure either, but I remember it was equivalent to this: *You eat infinitely many apples*.
>
> [Eve eats all the apples, and she and Adam are expelled from Eden.]
>
> Eve, furious. Why did you say God had allowed me to eat infinitely many apples?!?
>
> Serpent. Wait, I said it was *equivalent* to that. And it was. *You eat infinitely many apples v such that v ≠ BadApple* (*Y*) holds in the same worlds as *You eat infinitely many apples, period* (*Y**). One apple cannot make the difference between an infinite set and a finite one.

Now let *X* be *Eve did as she was told.* *Y*'s truth is wholly helpful to *X*, given that God had allowed Eve to eat infinitely many apples other than BadApple. There is just no disobedient way of doing that (see below for "ways"). Whereas *Y**'s truth is not wholly helpful to *X*, since there are ways for *Y** to hold that have Eve disobeying God.

7. MEREOLOGY

[21] Fine[2017]

What is it for X' to be $\leq X$ in $[P]$? You might think that $X' \leq X$ iff X implies, or necessitates, X'. But although this is how content-parts have often been understood, the view quickly runs into problems.[22] For one thing it allows X to be knocked out of proportion with Y by $X \vee S$, provided that S too is sufficient for Y. Which is surely the wrong result.

Socrates' drinking the hemlock (X) suffices, we assume, for his death (Y). X is proportional to the death only if nothing less suffices. Yet something less is bound to suffice, if $\leq$ is just the converse of implication. For let S be any other sufficient basis for death, say, falling off a high cliff. Then $X \vee S$ is a weaker sufficient condition for Y than X is. This disjunctive condition knocks X out of proportion with Y, if $\leq$ means is-implied-by. So $\leq$ had better mean more than that. The answer we'd like to give is that $X \vee S$, although *weaker* than X, is not *contained* in X. To be proportional to Y, X should have no proper *parts* sufficient for Y.

This notion of content-part is not available to the Humean, since it is hyperintensional—which is one more reason not to settle for Humeanism. Our approach to hyperintensionality, looking ahead a bit, will be in terms of *ways*. Conditions like X and X^*—*His praise is sung infinitely many days starting now* and *His praise is sung infinitely many days after 12019*—hold in the same worlds, but not always in the same ways in those worlds. No way of singing infinitely many days after 12019 involves singing tomorrow, but singing tomorrow may well have a role to play in how God's praise is sung infinitely many days starting now. Ways are the key as well to content-parts:

$X' \leq X$ iff

(i) every way for X to hold implies a way for X' to hold,

(ii) every way for X' to hold is implied by a way for X to hold.[23]

Ways bear too, it turns out, on the problem we are mainly concerned with in this paper, the problem of minimality. Details are given later. Suffice it for now to say that although the

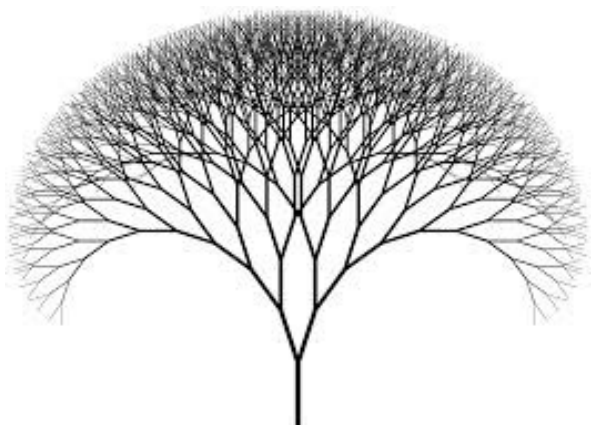
[22] Gemes[1994,1997],Fine[2013],Yablo[2014],Fine[2015a].

[23] Gemes[1994,1997],Yablo[2014],Fine[2015a],Yablo[2016],Fine[2017],

Humean Package faces multiple challenges, they all push in a similar theoretical direction.

8. BOTTOMLESS KINDS

A fractal is a geometrical figure containing isomorphic copies of itself; these will then contain isomorphic copies of *them*selves, and so on all the way down. An example is tree ***t*** below. Fractals are counterexamples par excellence to the minimality requirement. The fact that ***t*** exists ([***t*** *exists*], for short) is as helpful as it could be to *There are fractals*. You are not going to find a better candidate for a proportional, discerning, basis for the truth of *There are fractals* than the existence of ***t***.



A fact that is clearly out of proportion with *There are fractals* is [***t*** *exists and Sparky is a dog*]. What is the difference exactly? You can throw the Sparky conjunct out, of course, and still be left with a fact sufficient for the existence of fractals. But one can also throw out part of the fact that ***t*** exists. For the immediate right subtree ***u*** of ***t*** is also a fractal, and ***t***'s existence consists in the joint existence of ***u*** and ***v*** (***v*** is the rest of ***t***). It is not clear as yet why [***u*** *exists and* ***v*** *exists*] would be more proportional to *There are fractals* than [***t*** *exists and Sparky is a dog*], or for that

matter [***u*** *exists and Sparky is a dog*], given that the second conjunct is in each case dispensable.

Call a kind K bottomless if to be a K is to contain smaller Ks. If K is bottomless, then clearly, a minimal K is not to be expected. Are there other bottomless kinds, besides *fractal*?

A set is infinite iff all of its members can be paired off 1-1 with its members other than x, for some x in the set. Suppose that S is equipotent in that sense with $S_1 = S\backslash\{x\}$, and let y be a member of $S\backslash\{x\}$. Then if $y \in S_1$, it follows on standard assumptions that S_1 is equipotent with $S_2 = S_1\backslash\{y\}$, and so on without limit. *Infinite set* is thus a bottomless kind.

A property is *dissective* if a thing cannot instantiate it unless all its parts do.[24] This does not ensure bottomlessness all by itself, but it does if we add that proper parts always exist. Sellars uses the notion to illustrate his distinction between the "scientific" and "manifest" images of reality.

> Color expanses in the manifest world consist of regions which are themselves color expanses.[25]

The manifestly colored expanses form a bottomless kind for Sellars. (Of an especially pure sort. Fractals can contain non-fractals, but the parts of a blue expanse are all blue.) Aristotelian water is supposed to be dissective, and a stretch of continuous motion always subdivides into smaller stretches of continuous motion. A minimal verifier of *The particle was moving continuously at noon* is not to be hoped for.

Why do people think that X cannot be wholly relevant to Y, if less than X suffices for Y, when these problems are so obvious? A condition may retain its hold on us, it is true, even after we see that it cannot always be met. A set of everything is impossible, too, but that doesn't make it any less "what we wanted." Logicians *regret* the unavailability of a universal set; they look for

[24] Goodman[1966]

[25] Sellars[1963]

ways of approximating it or simulating it.[26] But this analogy takes us only so far; there is nothing to regret in the fact that we can't lay our hands on a minimal fractal.

9. SCHEMATIZATION

The Humean Package is more of a schema than a definite claim. Z is *causally* relevant to Y iff it extends to an X that *causally* suffices for Y, where nothing less causally suffices. Z is *ground*-relevant to Y (it is a difference-making ground, see section 3) iff it is part of a full ground X of Y such that nothing less than X fully grounds Y. Z helps to justify Y iff it is part of an X that fully justifies Y, but ceases to do so when anything is deleted. The "generic" notions of sufficiency ($\Rightarrow$), proportionality ($\propto$), and helpfulness ($\rightsquigarrow$) in [H] and [P] are really just shorthands for particular flavors $\Rightarrow^k$, $\propto^k$ and $\rightsquigarrow^k$ of these notions:

[$\mathbb{H}$] $Z \rightsquigarrow^k Y$ iff $Z \leq X$ for some (actual) X such that $X \propto^k Y$.

[$\mathbb{P}$] $X \propto^k Y$ iff (i) $X \Rightarrow^k Y$, (ii) for all $X' \leq X$, if $X' \Rightarrow^k Y$, then $X' = X$.

The superscripted "arrows" $\Rightarrow^k$, $\propto^k$, and $\rightsquigarrow^k$ stand ambiguously for the various kinds of sufficiency and relevance with which philosophers have concerned themselves: causal, logical, modal, nomological, explanatory, evidential, and so on.

Dividing things up in this way get us no closer to solving the minimality problem, but it

[26] This is part of the attraction of plural quantification.

clarifies the problem's shape and size. Take again *He is praised infinitely many days*. It has an unending chain of progressively weaker sufficers: he is praised every day from today on (X_0), every day from tomorrow on (X_1),…., every day from day n on (X_n), and so on. The sufficiency in this case is ground-flavored. Suppose we write $\Rightarrow^g$ for "is sufficient in the manner characteristic of a full ground for." Then we have

$X_0 \Rightarrow^g Y$

$X_1 \Rightarrow^g Y$

$X_2 \Rightarrow^g Y$

$X_3 \Rightarrow^g Y$,

....

$X_n \Rightarrow^g Y$,

.....

It follows from [ℙ] that $X \propto^g Y$ (X is proportional in the manner characteristic of grounds to Y) iff X has no proper parts X' such that $X' \Rightarrow^g Y$. But then, it holds of no X_i on the list that $X_i \propto^g Y$, since each X_i has a proper part X_{i+1} such that $X_{i+1} \Rightarrow^g Y$. Lacking a better candidate for proportional ground than these, we had better give up as well on locating a difference-making ground Z for Y. Difference-making grounds by [ℍ] will have to be parts of proportional grounds, and proportional grounds in the sense of [ℙ] just don't exist. So it will never be true that $Z \rightsquigarrow^g Y$ (Z will never be ground-relevant to Y), and it contributes nothing to the outcome of infinitely many praise days that we sing on any particular day(s) whatever.

The point is that we need to distinguish this variation on an old theme from other variations. This time let Y be *God is pleased at the amount of praise he gets,* and let the sufficiency be cause-and-effect-flavored (represented by $\Rightarrow^c$ rather than $\Rightarrow^g$). It holds of no X_i on the list that $X_i \Rightarrow^c Y$, since each X_i has a proper part X_{i+1} such that $X_{i+1} \Rightarrow^c Y$. And so it is never true, by the same reasoning as before, that $Z \rightsquigarrow^c Y$. God does wind up pleased if

praised every day, but not, it seems, due even in part to the praise received on any particular day(s).[27] Examples of the same sort can be given for any mode of relevance: moral, evidential, nomological, etc. The Humean Package—a theory $\mathbb{H}$ of relevance built on the back of a theory $\mathbb{P}$ of proportionality—needs fixing.[28] We will start, in section 11, with a particular *kind* of non-Humean proportionality from which the others can then hopefully be defined.

10. WAYS AND WORLDS

Parthood and proportionality (also "indifference" in section 12) are hyperintensional notions. To do them justice, we have to expand our toolkit, for "The possible worlds apparatus can only draw intensional, not hyperintensional, distinctions."[29] As it turns out, the role traditionally played by worlds is better played by *ways* (section 7),[30] and entities individuated by ways are hyperintensional right out of the box. P is true in the same worlds as $(P{\equiv}Q) \vee (P{\equiv}\neg Q)$, but the latter has different ways of being true.[31] Events that necessarily co-occur may have different ways

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[27] A case with fewer distractions: A buzzer sounds at the weigh station when a truck enters weighing over 70,000 pounds. The buzzer goes off "for no particular reason," judging by [$\mathbb{P}$] and [$\mathbb{H}$]. Reasons have to be drawn from conditions minimally causally sufficient for the effect, and there are no numbers minimally larger than 70,000.

[28] Our focus has been, and will continue to be, on $\mathbb{P}$. But $\mathbb{H}$ has issues of its own, which will come up briefly at the end.

[29] Berto[2017]

[30] See also Yablo[2014, 2017b].

[31] Assuming that the second can be true by way of P's truth and Q's falsity. This will be so on most accounts of truthmaking.

of occurring. A solid figure occupies most of the open sphere $\{<x, y, z> \mid x^2 + y^2 + z^2 < 1\}$ in a world *w* just if it occupies most of the closed sphere $\{<x, y, z> \mid x^2 + y^2 + z^2 \leq 1\}$. But these outcomes obtain in different ways. To occupy all of the closed sphere is a way of occupying most of the *closed* sphere, but not of occupying most of the *open* sphere.[32]

What is meant by a "way ω for *P* to hold"? I do not know how to define the notion, and will not even try. There is nothing scandalous about this. Does Lewis attempt to define "world *w* where *P* holds" when doing possible worlds semantics? Not at all. Two issues must be distinguished in his view:

(1) what is a "world in which proposition P is true"? and

(2) what is the "proposition P expressed by *P*"?

The first is trivial, if propositions are sets of worlds. P is true in a world *w* iff that world is a member of P. The second problem, that of associating propositions with sentences, is nothing special to do with worlds, and it is not made more difficult by the worldly conception. One approach is to let the intensions of atomic expressions be given outright; the intensions of complex expressions are then determined compositionally. Or we could determine atomic intensions using some kind of covariational metasemantics. Or we could approach the matter holistically, as Lewis does himself, using reference magnetism as a tie-breaker.[33]

The procedure works equally well if propositions are made up of non-worldly circumstances. *S* is true in circumstances *c* just if *c* belongs to S = the proposition that *S*. One can work with the

[32] Continuous motion occurs in the same worlds as discrete continuous motion*s*. But if a particle moves continuously from noon to one and then three to four, that is more of a way for continuous motion*s* to occur than continuous motion. (Thanks here to Kit Fine.)

[33] Lewis[1974],Lewis[1983]

first notion (propositional truth) today while leaving until tomorrow the problem of explaining how a sentence comes to express this proposition rather than that. This is a common strategy in semantics. Propositions for Kratzer are sets of situations. Propositions for Humberstone are sets of possibilities. Propositions for new-style expressivists like Yalcin and Moss are sets of probability-measures.[34] Propositions in truthmaker semantics are sets of ways.

11. THE "ONLY IN PART BY" TEST

Suppose I am right that way-for-it-to-be-that-*P* is on a par methodologically with world-in-which-*P*, and requires for present purposes no analysis. There is still the question of *which* unanalyzed notion is intended. Here my job is in one respect easier than Lewis's: "way for something *P* to be the case" is a more familiar and commonsensical notion than "world where *P* is true." But it is in another respect harder; for ways are a miscellaneous lot and I need to direct your attention to a particular sub-genre. Our target in the end is ways for it to be that *P*, but it helps to look more generally at ways for a thing *x* to φ. (Ways for it to be that *P* fall out as the case where *x* is a world and to φ is to be a *P*-world.)

So, let's try it. Here to get started are some paradigms of ways for *x* to do a certain thing, or instantiate a certain property:

Disjuncts: For *x* to sing is a way for *x* to sing or dance.

Instances: For *x* to sing is a way for something to sing.

Determinates: For *x* to yodel is a way for *x* to sing.

[34] Yalcin[2012], Moss[2018].

And here are some foils, that is, paradigms of failure to be a way.

Conjunctions: To sing and dance is NOT a way of singing.

Generalizations: For everything to sing is NOT a way for *x* to sing.

Manners: To yodel badly is NOT a way of singing.

Prequels: To dance is NOT a way of failing the course.[35]

Prerequisites: To dance is NOT a way of persisting over time.[36]

A few principles will help us to sort these cases out; we will see to a first approximation why the line is drawn where it is.

A first condition on ψ, if it is to count as a way of φ-ing, involves the notion of *only* φ-ing. Why is singing and dancing not a way of singing? A way of singing is not (even in part) a *further* thing one does, in addition to singing. Singing of its nature has to be done in some way or other (by yodeling, say), much as eating involves there being something or other that one eats. This is why to eat carrots, or yodel, is not to do a further thing besides eating, or singing. To sing and dance *is* in part to do a further thing, namely dance.[37] That one also dances means that one isn't only singing. Our first test, then, is

[35] The instructor disapproves of dancing; or, to dance cuts into your study time.

[36] Assuming that an instantaneous entity cannot dance.

[37] This idea is from Kratzer[1989].

ONLY-WAY To ψ is a way of φ-ing only if: *x ψ-d* is compatible with *x only φ-d*

Al yodeled is compatible, for instance, with *Al only sang.* But *Al sang* is likewise compatible with *Al only yodeled,* and singing is not a way of yodeling. One thing we could say here is that ψ-ing should necessitate φ-ing (as singing does not necessitate yodeling). But singing-while-persisting-over-time, which does necessitate singing, also passes the ONLY test. And we don't want to say that singing-while-persisting is a way of singing.

The difference is that one doesn't persist *by* singing, or for that matter sing by singing and dancing. Whereas one does sing by yodeling, and one may sing or dance by singing. And one arranges for something to sing, by arranging for *x* to do so.

BY-WAY To ψ is a way of φ-ing only if: *x* φ's *by* ψ-ing

One evening in 1905, Paula painted a still life with apples and bananas. She spent most of the evening painting and left the easel only to make herself a cup of tea, eat a piece of bread, discard a banana or look for an apple displaying a particular shade of red. Against the background of this situation, consider the following two dialogues that might have taken place the following day:

Dialogue with a Pedant

Pedant: What did you do yesterday evening?
Paula: The only thing I did yesterday evening was paint this still life over there.
Pedant: This cannot be true. You must have done something else like eat, drink, look out of the window.
Paula: Yes, strictly speaking, I did other things besides paint this still life. I made myself a cup of tea, etc

Dialogue with a Lunatic

Lunatic: What did you do yesterday evening?
Paula: The only thing I did yesterday evening was paint this still life over there.
Lunatic: This is not true. You also painted these apples and you also painted these bananas. Hence painting this still life was not the only thing you did yesterday evening. (608)

The pedant is technically correct, if a bore. But Kratzer rightly objects to the lunatic that Paula "didn't paint apples and bananas *apart* from painting a still life. Painting apples and painting bananas was part of her painting a still life" (608)

This is better, but still lets too much in. One can fail the course by dancing— when one ought to be studying—and to dance is not in the relevant sense a way of failing the course. If, as the story goes, the answer to "How do I get to Carnegie Hall?" is "Practice!" still practicing is not for us a way of getting to Carnegie Hall. The dancing is more like a cause, or facilitator, of — let us say *prequel* to — failing the course, and the practicing is a prequel to Carnegie Hall. Here is a third principle aimed at prequels:

WAY-IN To ψ is a way of φ-ing only if: *x* φ's *in* ψ-ing.

One may fail a course by dancing, but not (certain courses aside) in the act of dancing. No one gets in the act of practicing to Carnegie Hall. Singing and yodeling are different in this respect. Hank Williams sang not only by yodeling, but in the act of yodeling.

Only one of the foils remains to be dealt with. Yodeling badly is not supposed to be a way of singing. But can't a Hank Williams impersonator sing both by, and in, yodeling badly? My intuitions waver on this, but let's allow that it is possible. A different explanation will then be needed of why yodeling badly does not count as a way of singing. I know what I *want* to say: as long as Bert is yodeling, how well or badly he does it is irrelevant to whether he sings. This does not get us very far, though. For have no account as yet of (ir)relevance; the whole point of this paper is that (ir)relevance threatens to float out of reach once we see that it cannot be captured Hume-style in terms of minimality.

But what if the particular *type* of irrelevance now at issue was independently identifiable, without getting into grander issues about relevance as such? What gives me hope is that ways in the relevant sense are intimately related to *parts*; and relevance has a counterpart virtue on the side of parts that is easier to get a grip on. Examples (1)-(6) show how closely "to be *G* is part of being *F*" lines up with "to be $\bar{G}$ is a way of being $\bar{F}$."[38]

[38] *R* and *S* could be, for instance, *red* and *square*. $\veebar$ is exclusive disjunction.

(1) To be $\begin{pmatrix} R \\ \bar{R} \end{pmatrix}$ is $\begin{pmatrix} \text{part} \\ \text{a way} \end{pmatrix}$ of being $\begin{pmatrix} R \wedge S \\ \bar{R} \vee \bar{S} \end{pmatrix}$

(2) To be $\begin{pmatrix} R \vee S \\ \bar{R} \wedge \bar{S} \end{pmatrix}$ is NOT $\begin{pmatrix} \text{part} \\ \text{a way} \end{pmatrix}$ of being $\begin{pmatrix} S \\ \bar{S} \end{pmatrix}$

(3) To be $\begin{pmatrix} R \vee S \\ \bar{R} \wedge \bar{S} \end{pmatrix}$ is $\begin{pmatrix} \text{part} \\ \text{a way} \end{pmatrix}$ of being $\begin{pmatrix} R \veebar S \\ R \equiv S \end{pmatrix}$

(4) To be $\begin{pmatrix} R \supset S \\ R \wedge \bar{S} \end{pmatrix}$ is NOT $\begin{pmatrix} \text{part} \\ \text{a way} \end{pmatrix}$ of being $\begin{pmatrix} R \wedge S \\ \bar{R} \vee \bar{S} \end{pmatrix}$

(5) To be $\begin{pmatrix} R \supset S \\ R \wedge \bar{S} \end{pmatrix}$ is $\begin{pmatrix} \text{part} \\ \text{a way} \end{pmatrix}$ of being $\begin{pmatrix} R \equiv S \\ R \not\equiv S \end{pmatrix}$

(6) To be $\begin{pmatrix} R \equiv S \\ R \not\equiv S \end{pmatrix}$ is NOT $\begin{pmatrix} \text{part} \\ \text{a way} \end{pmatrix}$ of being $\begin{pmatrix} R \wedge S \\ \bar{R} \vee \bar{S} \end{pmatrix}$.

So, (1) to be red is part of being red and square, and its negation non-red is a way of being the negation of red and square, viz. non-red or non-square. (2) To be red or square is no part of being square, and to be neither red nor square is not a way of being non-square. (3) To be red inclusive-or square is part of being red exclusive-or square, and to be neither red nor square is a way of being red iff square. (4) To be non-red or square is not part of being red and square, while to be red and non-square is not a way of being non-red or non-square. (5) To be square if red is part of being square iff red, and to be red and non-square is a way of failing to be square iff red. (6) To be square iff red is NOT part of being square and red, and failing to be square iff red is not a way of being non-red or non-square.

To go by these examples, parts and ways are *duals* in something like the way box is dual to diamond. Much as φ holds necessarily iff ~φ fails to hold possibly, ψ-ing is part of φ-ing just if $\bar{\psi}$ - ing is a way of $\bar{\varphi}$ -ing. Turning this back to front and focussing on the direction of

interest,

PART-WAY To ψ is a way of φ-ing only if: to $\overline{\psi}$ is part of what is involved in $\overline{\varphi}$-ing.

Relevance may now be tested as follows. Suppose that ψ has an aspect that, by its irrelevance to φ, prevents ψ from being a way of φ-ing. This irrelevant aspect will show up under negation as an unwanted disjunct in $\overline{\psi}$ that prevents it from being a part of $\overline{\varphi}$.

Take the example of yodeling badly. Yodeling badly is a way of singing only if part of what is involved in *not* singing is to either not yodel at all, or else yodel well. To avoid yodeling may indeed be part of what it takes not to sing. But to yodel well, one must sing! So it is hard to see yodeling well as caught up (even disjunctively) in a part of not singing. To yodel badly is not, by our test, a way of singing. The test looks favorably on our paradigms, moreover. To sing remains a way of singing or dancing, since to do neither is in part not to sing. To yodel remains a way of singing, since not to sing is in part not to yodel. For Joe to sing remains a way for someone to sing, since part of what is involved in no one's singing is for Joe in particular not to sing.

Four conditions on ways have been suggested: ONLY-WAY, BY-WAY, WAY-IN, and PART-WAY. They constitute together the "ONLY IN PART BY" test for way-hood. I speak of a test, because the conditions do not pretend to get at what ways really are. They do seem however to correctly classify between them all the cases considered so far. A number of questions might be raised about them, and there are other conditions that might be considered instead, or as well. All of this is well worth pursuing, but not here. Our topic is relevance and it is time to get back to it.

12. INDIFFERENCE

God is pleased has an unending chain of progressively weaker sufficers: He is praised every day from today on, every day from tomorrow on, ..., every day after 12019, and so on. The weaker ones are no better, though, if all God wants is to be praised infinitely many days. There

would be something to regret in the sequence never terminating, if knocking off initial segments brought a feeling of progress, of getting closer to God's real reason for being pleased. But we never do get closer and nothing is ever gained.

Cantor was disappointed with what we now call the infinite numbers. No $\aleph_\alpha$ could satisfy him, because there was always a bigger one down the road, and bigger, in the infinity department, is better. A *truly* infinite number would be as large as possible. This is why he called the $\aleph_\alpha$s "transfinite," reserving "infinite" for a (putative) number too big for his system.[39] Cantor preferred larger numbers because they had more of what he wanted: size. If we perceive no advantage in X' (singing every day after 12019) as the cause of God's pleasure, over X (singing every day henceforth), it stands to reason that X' does not have more of what we wanted. It does no better proportionality-wise than X did, which means that X did no worse.

I want to focus on this idea that smaller is not necessarily better when it comes to proportionally causing Y—that X and X' are "the same to Y," in symbols, $X' \equiv_Y X$.[40] Now, we don't know what "same to Y" means as yet. If we did, we would take our existing account of proportionality,

[ℙ1] $X \propto^k Y$ iff (i) $X \Rightarrow^k Y$, (ii) for all $X' \leq X$, if $X' \Rightarrow^k Y$, then $X' = X$.

and replace the last bit of (ii) with $X' \equiv_Y^k X$, to obtain

[ℙ2] $X \propto^k Y$ iff (i) $X \Rightarrow^k Y$, (ii) for all $X' \leq X$, if $X' \Rightarrow^k Y$, then $X' \equiv_Y^k X$.

There are infinitely many so and so's is about size in the *how-many* sense, not the *inclusion*

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[39] Hallett[1986]. To continue with the (meaningless?) coincidences, Cantor was hospitalized for depression in 1899; *That Obscure Object of Desire*, or the novel it is based on, appeared the year before.

[40] Or, more carefully, since they could conceivably be the same to Y *as causes*, but different as, say, *grounds*, or *reasons*, $X' \equiv_Y^c X$.)

sense. That is why there is nothing to be gained proportionality-wise by knocking out one of the so and so's. *Y* doesn't *care* about, it is not *concerned* with, the kind of size where subsets are smaller. When we fix our attention on the *how-many* notion of size, we find that *X′* offers no advantages over *X*.

13. ABOUTNESS

A circumstance *Z* contributes^k to *Y* just if it is part of an *X* that is proportional^k to *Y*. *X* is proportional^k to *Y* if *Y* does not care about any differences that might obtain between *X* and those of its proper parts *X′* that also suffice^k for *Y*. (Our notation for this was $X' \equiv^k_Y X$.) *X* does not have to be minimal in all respects to be proportional to *Y,* the thought is, just the respects that matter, the ones *Y* is concerned about.

The "concern" is metaphorical, you'll be glad to hear. But the "about" and the "mattering" are not; they will be cashed out in terms of *ways of being true*. How is it that $P \vee \neg P$ is about a different matter than $Q \vee \neg Q$, when they are true in the same worlds? Well, they are true in different ways in those worlds. Why does the subject matter of $P \& Q$ include the subject matter of *P*, but not that of $(P \& Q) \vee R$, when $|P \& Q|$ (writing $|S|$ for the set of *S*-worlds) is a subset both of $|P|$ and $|(P \& Q) \vee R|$? Well, $(P \& Q) \vee R$ holds, sometimes, in ways not implied by any way for $P \& Q$ to hold; while the same cannot be said of *P* in relation to $P \& Q$.

This section attempts to make the notion of subject matter precise enough for the proposed application to "minimality in the respects that matter." We ask, first, what are subject matters considered as entities in their own right, and second, what is *the* subject matter of a particular sentence *S*?

A subject matter m— the number of stars, in Lewis's example— is given by specifying all the ways matters can stand where m is concerned. The ways they can stand number-of-stars-wise are for there to be no stars, or one star, or two stars, or etc. Formally we can think of m as a collection of set-of-worlds propositions P. m = {A, B, C,....} just if A, B, C,... constitute between them all the ways that matters can stand m-wise.

Subject matters can be more or less fine-grained. The number of stars is coarser-grained than which stars exist (henceforth the stars). It is finer-grained, though, than whether the number of stars is prime. m is as fine-grained as n when each n-cell subdivides into m-cells, and finer-grained when this holds in one direction only. The reader can check that this definition "works" if

the stars =

$$\{|\textit{Nothing is a star}|, |\textit{The only star is Sol}|,..., |\textit{The stars are Sol, Polaris, Vega,..}|,...\}$$

the number of stars =

$$\{|\exists_0 x\ star(x)|, |\exists_1 x\ star(x)|, ...|\exists_k x\ star(x)|,...\}$$

whether the number of stars is prime =

$$\{\bigcup_{prime(k)} |\exists_k x\ star(x)|, \bigcup_{\sim prime(k)} |\exists_k x\ star(x)|\}$$

We come finally to the subject matter of particular sentences. *S*'s subject matter s is made up of *S*'s various ways of being true; it is the set of all set-of-worlds propositions P such that *S* is true in way P in some world *w*. Assuming that *Stars exist* has a way of being true for each possible nonempty cast of stars, its subject matter will be what above we called the stars, except the first, star-less, cell must be dropped since *Stars exist* is false in that cell.

14. "EVERY BIT AS SUFFICIENT"

So: *Z* is relevant^k to *Y* just if *Z* is part of an *X* that is proportional^k to *Y*—an *X* no proper part *X'* of which "undercuts" *X* by sufficing^k for *Y* on a more economical basis. The question is, when *does* a still-sufficient proper part *X'* of *X* undercut *X* in this way? How indeed can *X not* be

undercut by X', if X' is every bit as sufficient for Y? Ways were supposed to be helpful here, but it remains to put the pieces together.

For X' to be "every bit as sufficient" as X for Y seems at first to mean that Y holds in as high a proportion of X'-worlds as X-worlds, viz. 100% of them. But a subtler reading is possible if statements holding in the same worlds can hold in a greater or lesser variety of ways.

Suppose that Alice has three children and Bert has two. Is Alice any more of a parent than Bert? Certainly it is no more *true* of Alice that she is a parent. But she has some sort of advantage parental-status-wise; for the truth of *Alice is a parent* is more thoroughly witnessed than that of *Bert is a parent*. The advantage is clearest if the witnesses to the one truth form a proper subset of the witnesses to the other; Alice has two children with Bert, and one not with Bert. Now Alice becomes, in addition to being more often a parent, more *richly* or *comprehensively* a parent than Bert is, since her status is witnessed by those who witness Bert's parental status and more besides.

This is how X can avoid being undercut by X', though Y is no less definitely true in X'-worlds than X-worlds: Y is not as richly provided for in X'-worlds as in X-worlds.

Take again *God is praised every day from now on*. It is better proportioned to *God is pleased* than *God is praised and dogs bark every day from now on*, because the effect is just as richly guaranteed whether dogs bark or not. Can we argue on a similar basis that *God is praised every day from tomorrow on* is better proportioned to *God is pleased* than *God is praised every day from now on*? No, for the effect is *not* as richly guaranteed as in worlds where the praise starts today. It is more richly guaranteed where the praise starts today, but not where dogs bark.

Given two statements P and Q both of which are true in w, let us say that P is as richly verified

in w as Q is if every way Q holds in w is also a way that P holds there. A is as richly verified (as verified, for short) as $A \vee B$ in worlds where B is false, but not worlds where it is true; since if B is true, then A is true in a proper subset of the ways in which $A \vee B$ is true.

A transworld version of this comparison suggests itself. Suppose we write $||P||_u$ for P's ways of being true in u. Then P is as richly verified in u as Q is in v just if $||Q||_v$ is a subset of $||P||_u$. And P is more richly verified if $||Q||_v$ is a proper subset of $||P||_u$. Thus *Goats eat cans* is as richly verified in u as *Goats eats cans or bottles* is in v, if goats eat cans in both worlds but stay away from bottles in v. The disjunction is more richly verified in v than *Goats eat cans* is in u, if goats eat bottles in v as well as cans.

Now we are ready to explain why a still-sufficient part X' of X sometimes, but not always, knocks X out of proportion with Y. The proposal is that X' undercuts X if it not only guarantees Y as *surely* as X does, but also as *fully* as X does. Let's write $\Uparrow^k Y$ for $\exists U\ (U \Rightarrow^k Y)$; in words, "$Y$ is guaranteed^k."[41]

[$\mathbb{S}$] X guarantees^k Y more fully than its part X' does iff

(i) $X \Rightarrow^k Y$ and $X' \Rightarrow^k Y$,
(ii) ($\exists X'$-world v)($\forall X$-worlds u) [$\Uparrow^k Y$ is not as richly verified in v as in u]

To see how this works, take again the *God is praised every day from today on* (X) and *God is praised every day from tomorrow on* (X'). X' and X both causally guarantee *God is pleased* (Y). But X guarantees^c Y more fully. For consider an X'-world v where the praise begins tomorrow. Is there an X-world u such that $\Uparrow^c Y$ is as richly verified in v as it is in u? There is not, for just the reason you'd think. Among the truthmakers in u for *God's pleasure is causally guaranteed* is the fact that his praise is sung every day from today on. Since this fact by hypothesis does not obtain in v, it cannot be the case in v that *God's pleasure is guaranteed* has all the truthmakers it has in u. Because X' guarantees Y less fully, it is no position to knock X out of proportion with Y.

[41] We'll consider later a slightly different reading.

Compare now *God is praised every day from tomorrow on* (X') with *God is cursed today and praised every day thereafter* (DX'). One certainly hopes that X' suffices as fully as DX' does for *God is pleased;* one hopes that for every X'-world v, *God's pleasure is causally guaranteed* is as richly verified in v as in some DX'-world u. Let v be a world where we rest today and begin praising God tomorrow. The question is, would his pleasure have been additionally guaranteed if we had cursed God today, instead of resting? The answer ought to be "no." For although we did not want X' to undercut X (= PX', where P says we praise God today), we do want it to undercut DX' (where, again, D says we curse God today). And to undercut DX', it must suffice as fully as DX' for God's pleasure.

15. ADDED GUARANTEES

Someone might object that God's pleasure *is* additionally guaranteed if we curse him today; for we have in DX' an additional guarantor. $\Uparrow^c Y$ is true, if we curse God today, not only on account of X'—our praising him from tomorrow on—but also on account of DX'— our cursing him today and praising him every day thereafter. Certainly DX' does not *intuitively* speaking add to the guarantee provided by X' for Y, the way PX' adds to the guarantee provided by X'. The question is whether we are entitled to this intuition. I say we are entitled to it, given a certain principle about truthmakers.

> *Parasite Principle:* Suppose that [E] is a truthmaker for S—a way for S to be true—and let [F]'s claim to truthmaker-hood be parasitic on that of [E]; that is, [F] makes S true only insofar as [E] did already, and because [E] did already. Then [F] is not a truthmaker for S.[42]

The problem with [*God's pleasure is causally guaranteed by our cursing him today and praising him every day thereafter*] as a truthmaker for *God's pleasure is causally guaranteed* ($\Uparrow^c Y$) is that it violates the Parasite Principle. The fact [F] expressed by

[42] Using "truthmaker" here, as elsewhere, for exact truthmakers. [F] may be an inexact truthmaker = something with an exact truthmaker as a part.

God's pleasure is causally guaranteed by DX'

verifies Y (*God's pleasure is causally guaranteed*), if it does, parasitically on the fact [E] expressed by

God's pleasure is causally guaranteed by X'.[43]

The fact expressed by

God's pleasure is causally guaranteed by PX' (= X)

cannot be accused of only parasitically verifying $\Uparrow^c Y$. The guarantee offered by praising God from today on (X) is self-standing, not a free rider on the guarantee offered by praising him from tomorrow on (X'). This is why DX' does not provide a further way for it to be true that $\Uparrow^c Y$, over and above X', while PX' does provide a further way, at least as far as the Parasite Principle is concerned.

Objection: Given that $\Uparrow^k Y$ was analyzed as $\exists U\ (U \Rightarrow^k Y)$, $\Uparrow^k Y$'s truthmakers should line up with the Us such that $U \Rightarrow^k Y$. Existential generalizations $\exists V \ldots V \ldots$ are made true quite generally by all facts of the form [$\ldots V \ldots$].

Reply 1: An existential generalization is certainly *witnessed* by each of its obtaining instances. But that is not to say that each instance has an equally good claim to the title of how and why the generalization holds. Take *There are dogs such that Sparky is a dog.* Is this made true by the fact that Daisy is such that Sparky is a dog? One wants to object Sparky must already be a dog, hence a dog such that Sparky is a dog, before Daisy can hope to achieve this status. Though clearly an instance of our generalization, the fact that Daisy figures in functions more as a lagging indicator of its truth than a basis for that truth. Likewise the fact that $DX' \Rightarrow^c Y$, though it guarantees that $\exists U\ (U \Rightarrow^c Y)$, derives any truthmaking powers it might possess from the prior fact that $X' \Rightarrow^c Y$. But then,

[43] X' says, as usual, that God is praised every day starting tomorrow.

by the Parasite Principle, $\Uparrow^c Y$ does not have $[DX' \Rightarrow^c Y]$ as an exact truthmaker.

Reply 2: Anyway $\Uparrow^k Y$ should probably not, in the end, be analyzed as $\exists U\,(U \Rightarrow^k Y)$, due to the difference in subject matter. $\Uparrow^k Y$ says of Y that it is guaranteed, while $\exists U\,(U \Rightarrow^k Y)$ says of the obtaining conditions U that at least one of them guarantees it.[44] *Y is guaranteed* stands to *Something guarantees Y* roughly as *The bucket is full* stands to *There is something whose parts occupy between them every bit of the bucket.* The quantified statement is verified, it might seem, not only by b (the water in the bucket) having parts that occupy every bit of the bucket, but also by $b+c$ having parts that do this, where $b+c$ is b summed with the Eiffel Tower. But it is hard to see in $b+c$ the materials for a further truthmaker for *The bucket is full*, over and above the truthmaker provided by b. Neither should we think that *God's pleasure is guaranteed* becomes more richly verified as random further conditions (praising Satan today, or cursing God) make themselves available to be fused with the infinite strings of praise days that constitute the statement's proper truthmakers.[45]

16. SUPER-HUMEANISM

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[44] Let P be the complex singular predicate "is a condition such that something suffices for it," and let Q be the complex plural predicate "are conditions of which at least one suffices for Y." Then *Y is guaranteed* predicates P of Y, while $\exists U\,(Z \Rightarrow_k Y)$ predicates Q of the *Us* (= all obtaining conditions). See Stalnaker[1977] for complex predicates.

[45] These judgments are supported as well by the ONLY IN PART BY test. Suppose that cursing God today and praising him thereafter was a way to arrange that God's pleasure was causally guaranteed. Why then does "I *only* saw to it that God's pleasure was guaranteed" seem to be undercut by "I saw to it that God was cursed and then praised, thus guaranteeing his pleasure"? Likewise I don't seem to arrange for that guarantee *by* arranging that God be cursed today and praised thereafter, given that God was pleased despite the cursing.

Hume's idea was to explain of proportionality as minimal sufficiency. Perhaps there is something right about this idea after all, *if* we are careful about what is being minimized, subject to which constraints. X' is in a position to undercut X, it begins to seem, if (but only if) it suffices for Y as fully as X does.

[ℙ3] X is proportional^k to Y ($X \propto^k Y$) iff

1. X guarantees^k Y ($X \Rightarrow^k Y$),
2. ($\forall X' \leq X$) [if X' guarantees^k Y as fully as X, then $X'=X$][46].

The super-Humean package is [ℙ3] plus

[ℍ] Z is helpful^k to Y ($Z \rightsquigarrow^k Y$) iff an X obtains such that $Z \leq X$ and $X \propto^k Y$.

Suppose, to vary the example, that Alice wins a prize (Y) if she moves for at least one hour between noon and two. Let X be her moving from 12.30 to 1.30 inclusive: her moving through the closed interval [12.30, 1.30]. How can X be proportional to Y, when X' — her moving through the open interval (12.30, 1.30) — is a proper part of X that also suffices?

The answer is that X suffices *better*; it guarantees Y more fully. For there are X'-worlds, like the world v where she moves precisely at times later than 12.30 and before 1.30, where Y is not as well provided for as it is in X-worlds. Here are some of the ways in which it is true in w that Alice moves so as to win the prize: she moves at all times t such that

[46] This is a version of [ℙ2] if "$X' \equiv^k_Y X$" is suitably unpacked. X' is no improvement on X (they're "the same to Y") if although weaker than X, it also guarantees Y less fully.

... $12.30 < t < 1.30$

... $12.30 < t < 1.30$ & $t \neq 1.00$

... $12.30 < t < 1.30$ & $t \neq 12.45$ & $t \neq 1.00$ & $t \neq 1.15$

... $12.30 < t < 1.30$ & $t \notin S$ (for some countable set S of times)

What is nice(r) about X-worlds is that Alice moves in them, not only in *these* prize-winning ways, but a bunch of additional prize-winning ways: for instance, at all times t such that

... $12.30 \leq t \leq 1.30$

... $12.30 \leq t \leq 1.30$ & $t \neq 1.00$

... $12.30 \leq t \leq 1.30$ & $t \neq 12.45$ & $t \neq 1.00$ & $t \neq 1.15$

... $12.30 \leq t \leq 1.30$ & $t \notin S$ (for some countable set S of times)

In general, $\mathcal{T}$-motion — moving at all times in $\mathcal{T}$ — is proportional to winning the prize just if $\mathcal{T}$ is a subset of $[12.00, 2.00]$ of measure 1. It follows by [$\mathbb{H}$] that $\mathcal{T}$-motion *contributes* to winning the prize just if $\mathcal{T}$ is a non-empty (possibly measure 0) subset of $[12.00, 2.00]$.

What can proportionality possibly mean, if minimal guarantors don't exist—if every X guaranteeing Y has a proper part that guarantees it just as surely? That question has now been answered. X is proportional to Y if X lacks proper parts that guarantee Y as *fully*. Equivalently X is the least $X' \leq X$ such that X' guarantees Y as fully as X does.[47] Z is relevant to Y if it is part of

[47] If it can happen that every X sufficing^k for Y has a proper part that still suffices^k for Y, shouldn't we be on guard too against the possibility that every X sufficing^k for Y has a proper part that suffices^k no

an X that is proportional to Y. That concludes the main task of this paper. But there is a bit more to say.

17. FORMS OF RELEVANCE

Mention was made at the outset of questions that had to be set aside. These were to do less with proportionality than the explanation of helpfulness in terms of proportionality:[48]

[ℍ] $Z \rightsquigarrow Y$ iff a fact X obtains such that $Z \leq X$ and $X \propto Y$.

Helpfulness so understood is *in-situ* and holistic. Whether Z contributes to Y depends very sensitively on what else is the case besides Z. A factor that is helpful to Y qua part of X_1 may work against it (it may be helpful to $\bar{Y}$) qua part of X_2, though of course X_1 and X_2 can't hold together. If one thinks that the party was good because it was fun— fun is by nature a good-maker—rather than fun being a good-maker derivatively, on this occasion—because of featuring essentially in a sufficient condition for goodness that happens to obtain— then [ℍ] is not going to satisfy you.

That is the first lacuna in our account. A distinction exists between in-situ helpfulness, on the one hand, and per se or presumptive or pro tanto helpfulness on the other, and we have not seen how to draw it. Second, helpfulness of the sort defined by [ℍ] is *factive*: Z is helpful to Y only if Y is really the case. But then, what of the idea of U holding *despite* Z? Z favors $\bar{U}$ here—or why say "despite"?— but the favoring cannot be factive, since $\bar{U}$ must fail, if U holds despite Z. The second lacuna is that Z can fight in a losing cause, and this remains unexplained. Nothing has

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less fully than X does? An argument by Zorn's Lemma (see the first Appendix) makes this unlikely.

[48] The superscripted k's are suppressed for readability.

been said in this paper about non-factive, defeated helpfulness. A lot has been written on these issues, especially in moral philosophy, by people like Ross, McDowell, and Dancy. I have no theory to offer, but would like to make a a suggestion: people interested in pro tanto relevance should be talking to people working on "ways" and subject matter (and vice versa). Appendix 2 shows how to tease apart various flavors of relevance with pretty much our existing machinery.

18. SUMMING UP

Helpfulness is a simple, deep, and elusive idea. Hume thought he had explained it with minimality, but the explanation didn't work, because some *X*s are helpful all the way down, A variant using *focussed* minimality—minimality where a certain subject matter was concerned— seemed to do better.

Subject matter relies on *ways of holding*, a notion with some of the features we were trying to explain, for instance, worlds being *relevantly* alike: alike on the score of how something is true in them. But we can in good conscience treat ways as primitive, by analogy with worlds (section 12)—especially since a lot of the work traditionally assigned to worlds is best done by ways (Fine[2015b,c],Yablo[2017b]).

Ways allow us to define the notion of *S* being as richly verified in this world as that, and thereby that of *X*'s part guaranteeing *Y* as fully as *X* does. *X* is proportional to *Y* iff its proper parts do not guarantee *Y* as fully as. *X* does. And *Z* is helpful to *Y* iff it figures in (is part of) a proportional *X*.

APPENDIX I

Suppose proportionality is explained as in [ℙ3]. Then we see in principle how X can defend its claim to proportionality with Y against a still-sufficient proper part X'. But does this really lay the problem to rest? If it can happen that every X that is sufficient for Y has a still sufficient proper part, perhaps it can happen too that every sufficient X has a proper part that suffices as fully as X does.

How would that work exactly? A proper part X' of X that sufficed as fully as X did was would have to hold in a certain kind of $\bar{X}$-world u: one where $\exists W\,(W \Rightarrow Y)$ was as richly verified as in v, for some X-world v. A world like u will not exist, though, if we load $|X|$ up in advance with all worlds of the relevant type—all worlds where Y is richly enough provided for. It follows from Zorn's Lemma that this is (under certain conditions) always possible.

Suppose we are given a partially ordered set $(\mathbf{K}, \ll)$. A subset $\mathbf{C}$ of $\mathbf{K}$ is a chain iff $\mathbf{C}$ is totally ordered by $\ll$, and bounded below iff $\exists \mathbf{x}\in\mathbf{K}\ \forall \mathbf{y}\in\mathbf{C}\ \mathbf{y}\ll\mathbf{x}$. $\mathbf{K}$ has a minimal element just if $\exists \mathbf{k}\in\mathbf{K}\ \forall \mathbf{j}\in\mathbf{K}\ (\mathbf{j}\ll\mathbf{k} \supset \mathbf{j}=\mathbf{k})$. We know by Zorn's Lemma that

(ZL) $\mathbf{K}$ has a minimal element if all $\mathbf{K}$-chains are bounded below.

But then, an X exists that is minimal among proper parts of X_0 no less sufficient (than X_0) for Y, provided that (*) holds:

(*) if $X_0 \geq X_1 \geq X_2 \ldots$, and X_i is no less sufficient for Y than X_0, then the X_is have a common part X_ω such that X_ω is no less sufficient for Y than X_0.

This is not implausible. It follows from two assumptions.

(1) If $\mathcal{X}_0 \subseteq \mathcal{X}_1 \subseteq \ldots$ are ways for things to be, then $\cup_k \mathcal{X}_k$ is a way for things to be

(2) If $\mathcal{A}, \mathcal{B}, \mathcal{C}, \ldots$ are ways for things to be, $\exists S$ ($\mathcal{A}, \mathcal{B}, \mathcal{C}, \ldots$ are S's ways of being true)

<u>Proof of (*) from (1) and (2).</u>

Let $X_0 \geq X_1 \geq X_2 \ldots$ and let $\mathcal{X}_\omega$ be $\cup_k |X_k|$. By definition of $\geq$, $\mathcal{X}_\omega$ is the union of all unions $\mathcal{X}^{\mathrm{C}}_\omega$ of chains of truthmakers $\mathcal{X}_0, \mathcal{X}_1, \ldots$ for $X_0, X_1, \ldots$ respectively. $\mathcal{X}^{\mathrm{C}}_\omega$ is a way for things to be by (1); and an X_ω exists with the $\mathcal{X}^{\mathrm{C}}_\omega$ s as its truthmakers by (2). X_ω is part of each X_k since each truthmaker $\mathcal{X}^{\mathrm{C}}_\omega$ for X_ω contains an $\mathcal{X}_i$. We need to show that X_ω is as sufficient for Y as X_0. This is immediate from the facts that

(i) each X_ω-world is an X_k-world for some k, and

(ii) $\exists W(W \Rightarrow Y)$ is, in each X_k-world u, as richly verified as in some X_0-world v. □

Given that (*) follows from (1) and (2), X_0 must on these two assumptions have a part X_ω that

cannot be further weakened without making it less sufficient for *Y*.

APPENDIX 2

Recall that *Z* is helpful *in situ* to *Y* iff an *X* holds that contains *Z* and is proportional to *Y*—that is, letting $\vec{Y}$ range over conditions proportional to *Y*, a $\vec{Y}$ holds that contains *Z*. But now, $\vec{Y}$ is still available to be quantified over whether it holds in this world or not. We can look then at the relation *Z* bears to *Y* if, for instance,

a) all $\vec{Y}$s contain *Z*

b) some $\vec{Y}$ s contain *Z,* and no $\vec{Y}$ contains $\bar{Z}$

c) in every *Z*-world, a $\vec{Y}$ holds that contains *Z*

d) in every world whatsoever, a $\vec{Y}$ *or* $\bar{\vec{Y}}$ holds that contains *Z* or $\bar{Z}$

A *Z* of the first type is, to have a word for this flavor of helpfulness, *indispensably* helpful to *Y.* A type-(b) *Z* is *intrinsically* helpful. A type-(c) *Z* is essentially helpful. A type-(d) *Z* is essentially *pertinent* to *Y.* We can distinguish as well various sorts of relative helpfulness:

e) $\vec{Y}$s containing *Z* are more numerous than $\vec{Y}$ s containing *Z′*

f) $\vec{Y}$ s containing *Z* are likelier than $\vec{Y}$ s containing *Z′*

g) $\vec{Y}$ s containing *Z* hold in closer worlds than $\vec{Y}$ s containing *Z′*

h) *Z* forms a larger part of $\vec{Y}$ s containing *Z* than *Z′* does of $\vec{Y}$ s containing *Z′*.

Z in the first case is more *often* helpful than *Z′*. In the others it is more *likely* helpful, or more

readily helpful, or more helpful simpliciter.

Sketchy as these proposals are, we have enough to try them out in a toy logical model. Outcomes Y will be sentences of propositional logic. Sufficiency is logical implication. Ways for things to be are conjunctions of literals (negated and unnegated atoms). A way $\vec{Y}$ for Y to be true is a conjunction of literals that implies Y and has no subconjunctions that imply Y.[49] One can show that

a) p is indispensably helpful to $p\&q$, but not to $p \vee q$ or $p \equiv q$

b) p is intrinsically helpful to $p \vee q$ and $p\&q$, but not to $p \equiv q$

c) p is essentially helpful to $p \vee q$, but not to $p\&q$ or $p \equiv q$

d) p is essentially pertinent to $p \equiv q$, but not to $p \vee q$ or $p\&q$

e) p is more often helpful than $\bar{p}$ to $p\&q\&r \vee \bar{p}\&\bar{q} \vee p\&\bar{q}\&s$[50]

f) neither of p, $\bar{p}$ is more likely helpful to $p\&q\&r \vee \bar{p}\&\bar{q} \vee p\&\bar{q}\&s$[51]

g) $\bar{p}$ is more readily helpful than $\bar{p}$ to $p\&q\&r \vee \bar{p}\&\bar{q} \vee p\&\bar{q}\&s$[52]

h) $\bar{p}$ is more helpful than p to $p\&q\&r \vee \bar{p}\&\bar{q} \vee p\&\bar{q}\&s$[53]

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[49] $\vec{Y}$ is in Quine's terms a *prime implicant* of Y (Quine[1955]).

[50] Since p figures in two ways for the disjunction to hold, while figures just in one.

[51] Since $p\&q\&r \vee p\&\bar{q}\&s$ and $\bar{p}\&\bar{q}$ are both 25% likely, if probabilities are assigned in the obvious way.

[52] Assuming that the middle disjunct would hold, were it the case that $p\&q\&r \vee \bar{p}\&\bar{q} \vee p\&\bar{q}\&s$.

[53] Since p takes up a third of $p\&q\&r$ and $p\&\bar{q}\&s$, while $\bar{p}$ takes up half of $\bar{p}\&\bar{q}$.

One can do *some* justice, then, to presumptive helpfulness in the present framework. And indeed one can pull apart different flavors of the notion. But other subtleties remain out of reach.

The elements of a proportional *X* must be in some broad sense relevant to *Y*. Helpfulness is a *kind* of relevance, but it is not the only kind. *X* may need to contain also facts which ensure of the helpful bits that they *are* helpful.[54] It may need to contain facts to ensure that the helpful bits *suffice*.[55] It may need to specify of potential spoilers that they did not materialize. (That my promise was not coerced is not a reason for lending you my car. Rather it is part of why it is not the case that the reason — my promise — was defeated.[56]) It may need to contain also threats to *Y*, if the events that blocked them were triggered by those very threats.[57] *X* may need to contain "intensifiers," trumps, switches, and other spin-control devices. These further devices are fascinating and important and beyond the reach of the present paper.[58]

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[54] Say, by noting that another potential route to *Y* was blocked (Yablo[2002]).

[55] Say, by clarifying the nature of *Y*'s demands: that Elsie is not a raven helps make the case for *All ravens are black* by showing it does not require Elsie to be black (Yablo[2014]:61ff , Skiles[2015]).

[56] Dancy[1983]

[57] Paul and Hall[2013]

[58] See Hawthorne[2002], Dancy[2004], Sartorio[2005], Sartorio[2006], Sartorio[2008], Leuenberger[2014], Skiles[2015], Skow[2016], Baron-Schmitt[2017], Mun˜oz[2017].

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