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Nominalism Through De-Nominalization[1]

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I. Introduction

Not all that long ago, second-order statements were thought to be committed twice over. The one commitment was to the members of a first-order domain D; the other was to the members of a second-order domain consisting of D's subsets and more generally the k-adic relations on D.

One sees this attitude, for instance, in Quine's description of second-order logic as "set theory in sheep's clothing."[2] What did Quine mean by this? His point could not have been that the *semantics* of second-order logic was set-theoretical. For the semantics of first-order logic is set-theoretical well; and Quine certainly didn't think that first-order logic was set theory in sheep's clothing.

Nor could the point have been that there are second-order statements whose *validity* goes with the truth-value of this or that set-theoretic hypothesis (for instance, the continuum hypothesis). After all, there are first-order statements too whose validity-status depends on how matters stand in the world of sets. If there are no infinite sets, then there are no infinite-domain models, and so the semantics finds

$$(0)\quad \forall x\,[x \neq s(x)\ \&\ o\neq s(x)\ \&\ \forall y\neq x\ s(y)\neq s(x)],$$

(essentially the first few Peano Postulates for arithmetic) to be unsatisfiable. It follows that the negation of (0) is valid unless some sets have infinitely many members. And once again, first-order logic is not, for Quine, set theory in sheep's clothing.

If the point was not that second-order semantics is set-theoretical, or that second-order sentences depend for their validity on the behavior of sets, what was it? Quine believed that second-order logic advanced *theses* which could not be *true* unless sets were counted into the range of their quantifiers. (Almost any second-order thesis has this property for Quine, because as he sees it, a second-order quantifier is basically a first-order quantifier stipulated to range over sets.) A statement that cannot be true unless Xs are counted into the range of its quantifiers is by *definition*—Quine's definition—ontologically committed to Xs. The second-order comprehension axiom (to take that example) needs to quantify over sets to be true, hence second-order logic is committed to sets. Practitioners of the logic of course inherit the commitment.

So much for what people *used* to think (and not unreasonably) about second-order statements. If the Quinean consensus has been breaking down, the credit goes to an observation of George Boolos's. Boolos noticed that many second-order quantifiers can be construed in terms of English phrases that carry *no* commitments not incurred already at the first order. Thus

(1) $\exists \mathrm{G}\ (\mathrm{G}a \ \&\ \neg \mathrm{G}b)$

can be interpreted as saying that

(2) There are some things such that *a* is one of them and *b* is not one of them.

Of course, it needs to be shown that "there are some things…" (a so-called plural quantifier) is committed just to the things and not their set. But a prima facie case is not hard to make. Suppose the second commitment were there. Then

(3) There are some things that are too many to form a set

would be self-defeating. It would be committed to entities of the very sort that it purports to reject. But on the contrary, (3) is an important and hard-won truth about sets. It is the clearest statement yet found of what separates the older, paradox-prone, concept of set from the (hopefully) non-paradoxical concept in use today.

II. Ontological Issues

Boolos's victory over the Quinean consensus has not been total.[3] The reason is that Boolos's scheme, effective as it is with *monadic* second-order quantifiers like $\ulcorner\exists \mathrm{G}\urcorner$ in (1), gets no grip whatever on *polyadic* second-order quantifiers like $\ulcorner\exists \mathrm{R}\urcorner$ in

(4) $\exists \mathrm{R}\ (\mathrm{R}ab \ \&\ \forall x\ \exists y\ \neg \mathrm{R}xy)$

One can't read the initial quantifier here as "there are some things…," because that leaves out the relationality; it makes no sense to say of "the things" that *a* bears them to *b*. Second-order logic however makes essential use of dyadic quantifiers like the one in (4). A famous example is the second-order formulation of equinumerosity, which occurs on the right hand side of "Hume's principle": the number of Fs = the number of Gs iff (∃R)(R is a one-one *relation* between the Fs and the Gs).

The almost universal response has been to look for ways of "coding" polyadic second-order quantifiers monadically. One idea is to construe dyadic ⌜∃R⌝ as a (disguised) monadic quantifier over pairs of domain elements. (4) could then be read as

(5) There are some pairs such that $\langle a, b\rangle$ is one of them, and for all *x* there is a *y* such that $\langle x, y\rangle$ is not one of them.

This approach is not *too* objectionable when one is working with a domain closed under the pairing operation—when doing second-order set theory, for instance. Even here, though, there is cause for unhappiness. "There are exactly as many even numbers as odd numbers" *looks* like a claim about (committed to) numbers and nothing else. But the second-order sentence that purportedly expresses the fact that there are as many evens as odds uses a dyadic quantifier; and so it is committed to ordered pairs on the proposed scheme. A lot of set-theoretical statements *do*, of course, carry a commitment to ordered pairs, construed as sets of the form $\{\{x\},\{x, y\}\}$. But that we're working in a theory that makes pairs *available* doesn't show that a sentence like "there are as many evens as odds" can *avail* itself of them at no cost to its intuitive meaning.

A stickier problem is that one uses second-order logic in connection with all *kinds* of domains, many of which have not much to offer in the ordered pair department. What if my intended subject matter is material objects, and only them? Do we have to treat my claim that "there are as many left shoes as right in my closet" as covertly concerned with pairs? It would be an enormous letdown if second-order statements, having just been cleared of the "serious" charge of being committed to all manner of sets, must plead guilty to the "lesser" charge of being committed to the special sets that are ordered pairs.

This stickier problem would not arise if pair-surrogates could be found in the first-order domain—say, the domain of material objects—to which we count ourselves already committed. (A version of the first problem would remain.) Lewis, Burgess, and Hazen take up the project of looking for concrete pair-surrogates in Lewis's *Parts of Classes*, "Appendix on Pairing." Because "natural" concrete codes for ordered pairs are not to be expected, this approach is bound to involve a certain amount of ad hocery (as the authors do not dispute). But that is not what bothers us.

Our concern is that the approach does not even get off the ground unless certain cosmological conditions are met. Lewis sums up the state of play in 1990 by saying that we need to have "infinitely many atoms [and] not too much atomless gunk" (1990, 121). Hazen in later work (1997) improves the condition to: there are infinitely many atoms or some atomless gunk. But the fact remains that depending on empirical circumstances, and on our chosen first-order domain, the trick may not work. This is discouraging. It is one thing to say that we can *arrange*, in fortunate conditions, for the *work* second-order languages do to be done without taking on any commitment to sets. It would be better if second-order languages were construable as *already* uncommitted to sets, and uncommitted regardless of the cosmological facts.

III. Why bother?

A word about motivation. "Nominalism" in our title refers not to the general ontological thesis—the rejection of abstract objects—but just to nominalism about second-order logic—the idea that second-order quantifiers need not be construed as ranging over abstract objects. *One* reason for being interested in nominalism about second-order logic is a prior nominalism about ontology. But there are other reasons, and it is not only the ontological nominalist who would (should) like to see second-order quantifiers interpreted in an ontologically unloaded way. Someone who embraces sets (concepts, properties…) does not necessarily think second-order statements quantify over them—not any more than a belief in angels requires one to think that second-order quantifiers range over angels. And someone who does think that second-order statements quantify over sets may still be interested in the expressive advantages offered by alternative construals.

Because there may be doubts on this score, consider some *applications* of the nominalizing project of potential interest to platonists. You may or may not be excited by them; that's not the point. The point is that the first two are of no use to anyone but a platonist, while the third is as much use to a platonist as to anyone else.

(A) Boolos suggests "there are some sets that are too many to form a set" as a restatement of the obscure "there are some would-be sets that are too big to be sets." The people whose doctrine Boolos is offering to clarify here are not nominalists. The nominalist doesn't think there are too many sets to form a set, because s/he doesn't think there *any* sets. In exactly the same way, it will be the platonist who benefits if we can reconstrue singular talk about class-sized relations as non-nominal talk about how things are related.

(B) "Fregean platonists" like Crispin Wright and Bob Hale attempt to deduce the existence of numbers from Fregean second-order logic plus definitions. An obvious objection is that since (Fregean) second-order quantifiers range over *concepts*, Wright and Hale are really only deriving one sort of abstract entity from another. How is that better than what Zermelo did when he reduced arithmetic to set theory? A construal of second-order quantifiers whereby they carried no new commitments would enable the Fregean platonist to answer this objection on its own terms.

(C) Advocates of Frege-style semantics say that predicates refer, but not to objects: that is, not to anything of the sort that a singular term can refer to.

This leads to the paradox of the concept "horse" and threatens to make the advocated semantical theory inexpressible. To say (what is literally true for Frege) that predicates do *not* refer escapes the paradox, but leaves us with no way to explain predicates' systematic contribution to truth-value. Maybe there is a middle way here. Systematic semantics has to *quantify*, but not necessarily *over* anything; we can do our semantics in a nominalistic second-order meta-language of the sort about to be explained. Examples (borrowing on the later explanation): "Susan is kind" is true because there is something that all and only the describable-as-"kind" things are, and that Susan is too. "Edinburgh is north of London" is true because things relate somehow such that "is north of" is true of *x* and *y* iff they are so related, and Edinburgh is so related to London.

IV. Grammatical Issues

One can think of Boolos's argument in the monadic case as having two steps. Step one is the identification of a non-committal-seeming bit of English. In step two we attempt to show that this bit of English is all that is needed to translate the relevant second-order quantifiers.

Existing attempts to extend the Boolos argument have focused entirely on the *second* step; they have tried to render the polyadic in plural terms. Our idea is to focus instead on the *first* step. Rather than trying to read everything in terms of the English plural, we should look for *another* English device, one that does better than the plural at capturing polyadic second-order meanings. First though let's look at another reason for dissatisfaction with Boolos's proposal.

The second-order quantifier is supposed to be a *predicative* quantifier. The positions it governs are built for predicative expressions; and it is predicative expressions one needs to "plug in" to obtain a grammatical substitution instance. Plural quantifiers have by contrast a distinctly *nominal* feel. To say "there are some things which…" feels like a way of talking about *things*, not what things are like or how they are related. This intuitive judgment is confirmed by the facts that

(a) "there are some things which…" binds nominal expressions like "they" and "them," and
(b) "there are some things which…" is completed by a verb phrase rather than a noun: "swim(s)" rather than "the swimmer."

One avoids the problem in practice by smuggling in appropriate connecting material. $\ulcorner\exists F\ Fb\urcorner$ says not that "there are some things such that they *b*" (!!) but that "there are some things such that they *include b*" or "…*b is one of* them." The $\ulcorner F\urcorner$ is seen as somehow sprouting the connecting material en route from its original position in the quantifier to its later position in the matrix; $\ulcorner F\urcorner$ is not born predicative, but has predicativeness thrust upon it. This shows that the informal practice, to the extent that it concerns itself with predicativeness at all, is without a consistent interpretation of the second-order variable. There appears, indeed, to be no grammatical way of reading both the plurality *and the predicativeness* back into the initial ⌜F⌝.

V. Connections

As we have seen, there are two ways in which Boolos's approach can seem insufficiently whole-hearted. These suggest in turn two desiderata, one ontological and one linguistic.

The first thing we want is a non-committal (or not-further-committal) English locution into which *polyadic* second-order quantifiers can be translated. The second is that that locution should respect the predicative, or at least *non-nominal*, character of these quantifiers, and for that matter monadic second-order quantifiers as well.

These desiderata, the ontological and the linguistic, are not as unrelated as they may seem. For a case can be made—*has* been made, by Quine in "On What There Is"—that adjectives and other non-nominal phrases are ontologically innocent. Speaking of McX's view that " 'There is an attribute' follows from 'There are red houses, red roses, red sunsets' " (1953, 10), Quine says that

> the word 'red'…is true of each of sundry individual entities which are red houses, red roses, red sunsets; but there is not, in addition, any entity whatever, individual or otherwise, which is named by the word 'redness'…McX cannot argue that predicates such as 'red'…must be regarded as names each of a single universal entity in order that they be meaningful at all. For we have seen that being a name of something is a much more special feature than being meaningful (10–11).

If predicates and the like needn't name to be meaningful—to make their characteristic contribution to truth-value—then we have no reason to regard them as presupposing entities at all. And this indeed appears to be Quine's view. But now he goes on to say something puzzling:

> One may admit that there are red houses, roses, and sunsets, but deny, except as a popular and misleading manner of speaking, that they have anything in common (10).

Quine is right, let's agree, that "there are red houses, roses, and sunsets" is not committed to anything beyond the houses, roses, and sunsets, and that one cannot infer that "there is a property of redness that they all share." But why should "they have something in common"—or better, "there is something that they all are"—be seen as therefore misleading? If predicates are noncommittal, one might think, the quantifiers *binding* predicative positions are not committal either. After all, the commitments of a quantified claim are supposed to line up with those of its substitution instances. Existential generalizations are *less* (or no more) committal than their instances, and universal generalizations are *more* (or no less) committal. "There is something that roses and sunsets are" is an existential generalization with "roses and sunsets are red" as a substitution instance. So the first sentence is no more committal than the second. But the second is not committed to anything but roses and sunsets. So the first isn't committed to anything but roses and sunsets either.

VI. Quantifiers

It's clear enough where Quine is coming from. To existentially generalize on "they are red" *would* be misleading if every quantifier was *nominal* in character; because then the generalization would have to be along the lines of "there is a property, viz. redness, that they have." But Quine gives no evidence that quantifiers are per se nominal. And in the present case the assumption seems clearly wrong, since one plugs in not the noun "redness" but the predicate "red."

Non-nominal quantification has not been much discussed by philosophers, with the shining exception of Arthur Prior. Here is what Prior says in "Platonism and Quantification" (chapter II of *Objects of Thought*):

> If we start from an open sentence such as "x is red-haired" and ask what the variable "x" stands for..., the answer depends on what we mean by "stands for." The variable may be said...to stand for a name (or to keep a place for a name) in the sense that we obtain an ordinary closed sentence by replacing it by a name, ...say, "Peter" ...the variable "x" may be said in a secondary sense to "stand for" individual objects or persons such as Peter. It "stands for" any such object or person in the sense that it stands for (keeps a place for) any name that stands for (refers to) an object or person. If we now consider the open sentence "Peter ϕ's Paul," it is equally easy to say what..."ϕ's" "stands for" in the first sense—it keeps a place for any transitive verb, or any expression doing the job of a transitive verb. The question what it "stands for" in the second sense...is senseless, since the sort of expression for which it keeps a place is one which hasn't got the job of designating objects...(35)

Does idiomatic English contain quantifiers governing variables like this—variables that don't (in the second sense) "stand for" anything? Prior believes it does:

> we form colloquial quantifiers, both nominal and non-nominal, from the words which introduce questions—the nominal "whoever" from "who," and the non-nominal "however," "somehow," "wherever," and "somewhere" from "how" and "where"[4]...no grammarian would count "somehow" as anything but an adverb, functioning in "I hurt him somehow" exactly as the adverbial phrase "by treading on his toe" does in "I hurt him by treading on his toe"...What is [also] done in English [when a non-nominal quantifier is needed] is simply to extend the use of the "thing" quantifiers in a perfectly well-understood way, as in "He is something that I am not—kind"..."something" here is quite clearly adjectival rather than nominal in force. (37)

According to Prior, whether we quantify nominally or non-nominally makes all the difference where ontological commitment is concerned. A propos "I hurt him somehow," he remarks that

> we might also say "I hurt him in some way," and argue that by so speaking we are "ontologically committed" to the real existence of "ways"; but…there is no *need* to do it this way…(37);

the implication is that if we *don't* do it this way, we take on no more commitment to ways than would be incurred with "by treading on him." Similarly, we *could* say "he exemplifies something that I don't: kindness," in which case we are prima facie committed to properties. But there is no need to do it like that. If we stand pat with adjectival "something," we incur no more commitment than we would with an adjective like "kind."

VII. Commitment

The claim is that non-nominal quantifiers—quantifiers like "somehow" and adjectival "something"—carry no commitments. What is the evidence for this? Our first argument (to which we attach the least weight) is implicit in the quotations from Prior.

> *Argument from Instances*: Use of a quantifier commits one at most to entities of the kind referred to by the phrases its bound variables stand in for.[5] The phrases a non-nominal variables stand in for—phrases like "by treading on him," and "kind"—do not refer at all.[6] So non-nominal quantifiers carry no commitments.

A second argument tries to do without any assumptions about the semantics of quantifiers:

> *Argument from Entailment*: Suppose that "I hurt him somehow" were committed to entities beyond those presupposed by "I hurt him by treading on him," that is, me and him and (maybe) my foot. Then "I hurt him somehow" would not be trivially entailed by "I hurt him by treading on him"—because it is not a trivial matter whether these additional entities exist. "I hurt him somehow" *is*, however, trivially entailed by "I hurt him by treading on him." So there is no additional commitment. Likewise, "he is something that I am not" follows trivially from "he is kind and I am not." The inference would not be trivial if "he is something that I am not" were committed to entities other than him and me. So it isn't committed to entities other than him and me.

A third argument is adapted from Boolos.

> *Argument from Consistency*: Suppose that the "something" in "*a* is something that *b* is too" carried a commitment to BLAHs—properties, say, or sets. Then to say "*a* is something that *b* is too, but there are no properties or sets to witness the fact" would be self-undermining. And in general it isn't. Sometimes indeed the claim is importantly true: "*a* is something that *b* is too, viz. not a member of itself, but as we know from Russell's paradox there is no witnessing set." Likewise it is quite consistent to say that "*a* and *b* are related somehow, in that *a* is a member of *b*, but the things so related are too many to fit into a set."

Our final argument tries to make use of the "reason" why Russell's contradiction arises:

> *Argument from Cardinality*: By Cantor's theorem, every domain contains objects *x*, *y*, *z*,…such that no domain element contains all and only those objects. Another way to put it is that the following is mathematically impossible:
>
> (i) take any objects you like, there's an object containing them and nothing else.
>
> But it is not at all impossible—it is on one reading quite *true*—that
>
> (ii) take any objects you like, they are something that the rest of the objects are not.
>
> A die-hard objectualist might try to construe the "something" in (ii) in terms of container-objects somehow eluding the grasp of the initial "any objects you like." But this escape hatch can be closed by stipulating that the initial quantifier is absolutely universal. Not only does this stipulation fail to make (ii) look any less consistent, (ii) continues to look *true*. It could not be true on the stipulated reading if "something" had ontological import.

VIII. The Interpretation

Non-nominal quantifiers allow for something like anaphoric cross-referencing—the kind we see in "That car is mine, but you can use *it*" and "Someone came into the store, and *she* demanded satisfaction." The reason for the hedge is that cross-*reference* is a privilege reserved to referring, so presumably nominal, phrases. Non-nominals do, however, allow for anaphorical cross-*indexing*. This was pointed out years ago by Nuel Belnap and Dorothy Grover:

> Anaphors do not always occupy nominal positions. There are, for example, proverbial uses of 'do.' 'Do' is used as a…quantificational proverb: "Whatever Mary did, Bill did," "Do whatever you can do." "Such" and "so" can be used anaphorically

as proadjectives: "The pointless lances of the preceding day were certainly no longer such" (Scott), "To make men happy and to keep them so" (Pope) (Grover 1992, 83–4).

Our strategy will make use not of pro-verbs or pro-adjectives but pro-*adverbs*, such as *likewise* in "I despise you, and the boss feels likewise," and *thus* in "he did it by breaking the window, and we did it thus, too," and (especially) *so* in "they are related as brother and sister, and we are so related as well."

Suppose we paraphrase "Connecticut is larger than Delaware" by the more cumbersome "Connecticut is related to Delaware in that the former is larger than the latter." Then we can say that

(6) Connecticut is related to Delaware in that the former is larger than the latter; Texas is so (thus, likewise) related to Nebraska.

From (6) it follows that

(7) Connecticut is related to Delaware somehow such that Texas is so related to Nebraska.

This gives us enough to begin thinking about how to render second-order ⌜∃R…R*xy*…⌝ . A natural thought given (7) is: ⌜something is related to something somehow such that…*x* is so related to *y*.⌝ This has two problems, however, one logical and one having more to do with readability.

The logical problem (to put it in objectualist terms) is that not *every* way of being related is such that things are in fact related in that way. The proposal in other words overlooks the empty relation. An example of the trouble this causes is that the logical truth ⌜$\exists R\ \forall x\ \forall y\ \neg Rxy$⌝ is mapped onto the obvious falsehood that 'something is related to something somehow such that no objects are so related.'

Boolos faced a similar problem when constructing his translation scheme in terms of plurals; the most straightforward approach takes ⌜$\exists P\ \forall x\ \neg Px$,⌝ a second-order logical truth, to the falsehood 'there are some things such that nothing is one of them.' Boolos's solution was to toss in a special-purpose disjunct for the empty case: ⌜∃P ---Px---⌝ goes to ⌜there are some things such that ---x is one of them---, *or else* ---x ≠ x---⌝.

It would be easy enough for us to follow his lead. But, at the risk of getting ahead of ourselves, this would be to throw away one of the nicer features of our approach. Non-nominal quantifiers differ from plural quantifiers in being open to an *intensional* interpretation; the things that *are* thus and so related may or may not be the ones that *would* have been so related had matters been otherwise. (See section X.) A special case of this is that things *could* have been related somehow such that nothing is so related in actual fact. When we run the formula ⌜$(\exists R)[\neg(\exists x)(\exists y)Rxy\ \&\ \Diamond(\exists x)(\exists y)Rxy]$⌝ that ought to express this possibility through the envisaged interpretation scheme, it winds up affirming instead the possibility of objects distinct from themselves.

A different line on the empty-relations problem starts by noting that quantificational adverbs can "reach inside" negation contexts and attach themselves to the negated verb. One says of a world traveler that "there must be *somewhere* he hasn't been," meaning by this not the negation of "he's been somewhere," but the existential generalization of "he hasn't been *to Tasmania*." Just so, one might say (of people standing in surprisingly many relations) "surely they are not related *somehow*," and mean, not the negation of "they are related somehow" (that would be silly), but the existential generalization of "they are not related *as brother and sister*."

All of that granted, we can interpret $\ulcorner \exists \mathrm{R}$ ---Rxy---$\urcorner$ as $\ulcorner$something is-or-isn't related to something somehow such that ---x is so related to y---$\urcorner$. Now the logical truth $\ulcorner \exists \mathrm{R}\ \forall x\ \forall y\ \neg \mathrm{R}xy \urcorner$ goes into the English truth that something is-or-isn't related to something somehow such that no objects are so related. So the logical problem—the one brought on by empty relations—is solved.

Our solution, however, exacerbates the readability problem. $\ulcorner \exists \mathrm{R}$ ---Rxy---$\urcorner$ we are reading as $\ulcorner$something is-or-isn't related to something somehow such that…$\urcorner$ It's distracting to have to plough through seven off-topic words before reaching the "somehow" that was after all our reason for coming. Why are we putting *objects* front and center, when our real concern is not with them but the way they are (or are not) related? Moving the "somehow" to the front, as in "*somehow* an object is-or-isn't related to an object…," helps a little, especially with a Yiddish intonation pattern: "*somewhere* you left it, that's all you can say?" Another thing that helps is to *compress* "an object is-or-isn't related to an object" to "things relate." Putting these two suggestions together, "an object is-or-isn't related to an object somehow…" is abbreviated to "somehow things relate …" This lets us interpret dyadic second-order logic as follows:

(a) $\mathrm{Tr}(\ulcorner \neg \phi \urcorner) = \ulcorner$it is not the case that$\urcorner \frown \mathrm{Tr}(\ulcorner \phi \urcorner)$
(b) $\mathrm{Tr}(\ulcorner \phi\ \&\ \psi \urcorner) = \mathrm{Tr}(\ulcorner \phi \urcorner) \frown$ 'and' $\frown \mathrm{Tr}(\ulcorner \psi \urcorner)$
(c) $\mathrm{Tr}(\ulcorner \exists x_i\ \phi \urcorner) = \ulcorner$something$_i$ is such that$\urcorner \frown \mathrm{Tr}(\ulcorner \phi \urcorner)$
(d) $\mathrm{Tr}(\ulcorner \exists \mathrm{R}_i\ \phi \urcorner) = \ulcorner$somehow$_i$ things relate such that$\urcorner \frown \mathrm{Tr}(\ulcorner \phi \urcorner)$
(e) $\mathrm{Tr}(\ulcorner \mathrm{R}_i(x_j, x_k) \urcorner) = \ulcorner$it$_j$ is so$_i$ related to it$_k\urcorner$

According to the scheme defined by (a)–(e), $\ulcorner \exists \mathrm{R}\ \forall x\ \exists y\ \mathrm{R}xy \urcorner$ says that

(8) Somehow things relate such that everything is so related to something.

The reader can try his/her hand at interpreting other dyadically quantified formulae; no special difficulties arise, once one gets past the initial awkwardness of using "somehow things relate" to express not that things *are* so related but their classifiability as so related or not.

So much for the dyadic case. *N*-adic quantifiers with $n>2$ present no additional problems; the sort of cross-indexing used above works with them too.

Example: Utah is intermediate in size between Nevada and Colorado, and Alabama is intermediate between Georgia and Mississippi. So, Utah, Nevada, and Colorado are related somehow such that Alabama, Georgia, and Mississippi are so related as well.

IX. Monadic Quantifiers

Now for the case that Boolos started, and finished, with: the case where $n=1$. Note before we begin that it wouldn't be *too* much of a problem if no suitable translation of monadic ⌜∃P ---P*x*---⌝ could be found. This is because (speaking like an objectualist) for any P, there's a relation R such that the Ps are the things that bear R to themselves; and vice versa. Thus one can always mimic the effect of monadic ⌜∃P ---P*x*---⌝ with a diagonal dyadic construction ⌜∃R ---R*xx*--- .⌝ But while it's nice that we are ready with an excuse if monadic second-order quantifiers should prove untranslatable, it would be nicer if we could just go ahead and translate them.

Picking up on the discussion above of adjectival "something," one idea is to read ⌜$\exists P_i$ ---$P_i x_j$---⌝ as ⌜an object is something$_i$ such that ---that$_i$ is what it$_j$ is---⌝. The translation of ⌜$\exists P\ \forall x\ (Px \rightarrow Qx)$⌝ would be ⌜an object is something$_i$ such that a thing is that$_i$ only if it is also Q⌝, or more colloquially, ⌜an object is something$_i$ that$_i$ only Qs are⌝.

If we want a treatment more in keeping with the dyadic case, a verb like 'determined' can be used, on the understanding that to be determined P-ly is the same as being P.[7] Should each of two objects be red, we will say that ⌜*a* is determined redly, and *b* is so determined too⌝. Next we introduce ⌜*a* is determined somehow⌝ as standing to ⌜*a* is determined redly⌝ just as ⌜*a* and *b* are related somehow⌝ stood to ⌜*a* and *b* are related in that *a* is larger than *b*.⌝ Our formula ⌜$\exists P\ \forall x\ (Px \rightarrow Qx)$⌝ now translates as ⌜somehow$_i$ an object is determined such that only Qs are so$_i$ determined⌝.

X. Interactions with Plurals

Whenever it makes sense to say, in the singular, that ⌜*a* and *b* are related somehow$_i$⌝ and ⌜it$_j$ is so$_i$ related to it$_k$⌝, it can also be said, in the plural, that ⌜the Fs are related to the Gs somehow⌝ and ⌜they$_j$ are so$_i$ related to them$_k$⌝. ('The soldiers are somehow related to the students—they have them surrounded—and the students are likewise related to the administrators.') This suggests that the translation scheme in **VII.** should extend to second-order descriptions of pluralities.

Begin by adding to the formal language *first*-order plural variables $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$, ... — not to be confused with our existing *second*-order variables P_1, P_2, P_3, ...!!!! Taking a leaf from Boolos, we read ⌜$\exists \mathbf{x}_i ... y_j\, \varepsilon\, \mathbf{x}_i ...$⌝ as ⌜there are some things$_i$ such that...y$_j$ is one of them$_i$ ... ⌝.[8] Next come *second*-order plural variables $\mathbf{R}_i$. These function grammatically as predicates taking first-order plurals ('the students') as arguments; so ⌜$\mathbf{R}_i\mathbf{x}_j\mathbf{y}_k$⌝ has the grammar of 'the soldiers have the students surrounded'. The translation rule, finally, is

(f) $\mathrm{Tr}(\ulcorner \exists \mathbf{R}_i \urcorner) = \ulcorner$somehow$_i$ things relate such that…$\urcorner$
(g) $\mathrm{Tr}(\ulcorner \mathbf{R}_i \mathbf{x}_j \mathbf{y}_k \urcorner) = \ulcorner$they$_j$ are so$_i$ related to them$_k$$\urcorner$

where the right-hand side of (f) is an abbreviation-for-readability of $\ulcorner$things are related (or not) to things somehow$_i$ such that…$\urcorner$ The elaborated scheme takes

(9) $\exists \mathbf{R}\ \forall \mathbf{x}\ \exists \mathbf{y}\ \mathbf{Rxy}$

to

(10) Somehow$_i$ things relate such that, take any objects$_j$ you like, there are objects$_k$ to which they$_j$ are so$_i$ related.

Why should anyone *care*, though, about the possibility of combining plurals with non-nominals in this way?

One advantage of combining them is that it helps us to disentangle two distinctions: singular vs. plural and first-order vs. second-order. Some, over-influenced by Boolos, have come to see these distinctions as one and the same. According to us, they cut across each other in every possible way:

variable///substituend	singular	plural
first-order	*x*///‘17’	**x**///’the prime numbers’
second-order	P/// ‘…is prime’	**P**/// ‘…are co-prime’

A second advantage of mixing plurals in is that it helps us to fend off an unintended interpretation of $\ulcorner \exists \mathrm{P} \urcorner$—the one that says that $\ulcorner \exists \mathrm{P}\ (\mathrm{P}a\ \& \mathrm{P}b) \urcorner$ is true only if *a* and *b* have something “nice” in common; they are both green, say, as opposed to both being grue. Such an interpretation is bound to invalidate the claim that

(11) $\forall \mathbf{x}\ \exists \mathrm{P}\ \forall y\ (\mathrm{P}y \leftrightarrow y\ \varepsilon\ \mathbf{x})$,

that is,

(12) Take any objects$_i$ you like, there is something$_j$ that they$_i$ and only they$_i$ are.

One can secure the same result for polyadic second-order quantifiers if the background theory has ordered tuples; just lay it down that

(13) Take any n-tuples$_i$ you like, things relate somehow$_j$ such that $x_1 \ldots x_n$ are so$_j$ related iff $\langle x_1, \ldots, x_n \rangle$ is one of them$_i$.

Can plurals be used to fend off the "nice" interpretation even in the absence of n-tuples? If they can, we haven't been able to figure out how. It should be stressed, however, that that *is* the intended result; there is no reason why non-nominal second-order quantifiers should not cover the same extensional ground as gets covered on the familiar nominal interpretation. How much ground *does* get covered is controversial, of course. The point is just that non-nominal quantifiers are plausibly in the same boat as nominal ones. (See section X.)

A third advantage is that we get a cleaner formulation of the Argument from Cardinality (section VI). Suppose we were to construe ⌜∃P⌝ in (11) as an objectual quantifier over, say, sets. Then the meaning of (11) would be given not by (12) but

(14) Take any objects you like, there is a set containing exactly them.

And we know that (14) is false; it's in direct contradiction with Cantor's theorem. So to the extent that (11) seems unproblematic, an objectual interpretation of ⌜∃P⌝ cannot be right.

A fourth advantage of combining plurals with non-nominals is the gain in expressive power. This is suggested already by (11), but a clearer case is

(15) $\neg\exists \mathrm{R}\ \forall \mathbf{y}\ \exists x\ \forall z\ (z\ \varepsilon\ \mathbf{y} \leftrightarrow \mathrm{R}xz)$,

which says that the objects cannot be paired off with the pluralities—in effect that there are more pluralities of objects than objects. [The objects are outnumbered by the things objects can be.] This could be expressed just with plural quantifiers if we had ordered pairs, but more complicated examples can be given where the combination of plurals with non-nominals seems essential even granted the ordered pairs. One can "say," e.g., that there are more pluralities of pluralities than there are pluralities.

(16) $\neg\exists \mathbf{R}\ \forall \mathbf{P}\ \exists \mathbf{x}\ \forall \mathbf{y}\ (\mathbf{R}(\mathbf{x},\mathbf{y}) \leftrightarrow \mathbf{P}(\mathbf{y}))$.

Here are some more attempts to say it in English (choose the one you dislike the least): The things objects can be are outnumbered by the things *they*—the things objects can be—can be. There are more things that objects can *collectively* be than things objects can *distributively* be. Just as it is not the case that there *is an object* for each thing objects can distributively be, it is not the case that there *are objects* for each thing objects can collectively be.[9]

A fifth advantage of mixing plurals in with non-nominals is that where plurals are rigidly extensional—if x is one of them, it is one of them necessarily—non-nominals are flexible as between extensional and intensional readings (see the next section). The combination thus provides a natural setting in which to study interactions between extensional and intensional styles of classification.

XI. Comparison with the standard semantics

The "standard semantics" for second-order languages is extensional; it never happens that second-order variables are assigned different values despite being true of the same things. Nothing much *hangs* on the extensionality assumption; one could equally construe the second-order variables as denoting intensional entities that are finer-grained than their extensions. That having been said, however, one might wonder whether the non-nominal approach stands on the question of extensional vs. intensional.

You may say that the question makes no sense. One can't discuss the identity-conditions of the values of second-order variables unless the second-order variables *have* values. And on the non-nominal interpretation, they don't.

But not so fast. The question of extensionality is really a question about whether variables true of the same objects are thereby identical in their total contribution to truth value. Are second-order variables (as interpreted here) extensional in this sense?

It all depends; one has to look what other semantic machinery is present in the language. Usually in discussions of these matters, we are thinking of ordinary second-order predicate calculus unsupplemented with any clever devices. If this is the sort of language at issue, then which objects P is true of does indeed fix its semantic potential. Our "semantics" is fully extensional.

But suppose the language contains modal operators. Then it's compatible with everything we've said that predicate-variables true of the same objects should fail to be intersubstitutable in all contexts. It could happen, for instance, that

(17) $\exists \mathrm{P}\, \exists \mathrm{Q}\, [\forall x\, ((\mathrm{P}x \leftrightarrow \mathrm{Q}x)\ \&\ \Diamond\, \exists y\, (\mathrm{P}y\ \&\ \neg \mathrm{Q}y))]$,

comes out true, because

(18) There is something$_i$ that my cat is—a creature with a kidney—and something$_j$ that my dog is—a creature with a heart—such that everything that is that$_i$ is that$_j$ and vice versa; but there *could* be a thing that was that$_i$ and not that$_j$

Admittedly, it's *also* compatible with everything your typical *objectualist* says that (17) should come out true. Your typical objectualist, after all, has no particular view about how to deal with second-order quantification into modal contexts.

She *might* take the position that the values of predicate-variables should continue to be sets when modal operators are introduced; and she might think that sets have their members essentially; and she might conclude from all this that (17) is false. But she might equally think that when the logic goes modal, we should take the variables to stand for intensions (functions from worlds to extensions); in that case (17) will almost certainly come out true, since intensions which coincide on one world need not coincide on others. Either way, if she is extensionality-minded, she will probably want to lay it down as an axiom that

(19) $\forall P \,\Box\, \forall x\,(Px \leftrightarrow \Box Px)$

—because (19) is true in all models, if she takes the first view, or because she wants to restrict attention to the models that make (19) true, if she takes the second. Be all that as it may, from (19) it follows, given the usual sorts of assumptions, that the extensionality scheme

(20) $\forall P\ \forall Q\ [\forall x\,(Px \leftrightarrow Qx) \rightarrow (\ldots P\ldots \leftrightarrow \ldots Q\ldots)]$

holds in full generality, that is, for modal and non-modal contexts alike. Depending on the application, of course, the assumption of extensionality may be unwelcome and out of place. But in that case the objectualist is free to simply refrain from imposing (19) or any similar condition.

The reason for mentioning all of this is that the *non*-objectualist would seem to be in exactly the same position. If and when she is attracted to extensionality, she can lay it down that (19). It is true that (19) means something different in her mouth; it means, simplifying some, that

(21) whatever$_i$ a thing is, it is necessarily, and vice versa.

But the effect is the same. (19) assures the non-objectualist too that extensionality reigns. Should she encounter an application where extensionality is not wanted, she is just as free as the objectualist to ditch (19), thus opening the door to an object being something that it might not have been.

A second and more controversial feature of the "standard semantics" for second-order languages is that it makes use only of *full* models, that is, ones whose second-order domain contains *every* subset of the first-order domain D, and for n-adic quantifiers every subset of D^n. This is what gives standardly interpreted second-order languages their stunning logical power: the power to pin down the standard model of arithmetic, for example, and to settle the truth-value of the continuum hypothesis. It is also what makes some commentators wonder what in our thought and practice could possibly rule out non-standard interpretations. Couldn't our quantifiers be ranging over an *approximation* to all the subsets such that creatures like ourselves could form no notion of what was missing?

Someone might say: your acceptance of (12), which says that given any objects there is something that they alone are, suggests that you are committed to a standard-*like* interpretation in which (to put it a bit paradoxically) $\ulcorner \exists P \urcorner$ ranges over *all* the things that domain elements can be.

Not that the commitment would be so unwelcome, does (12) really force it on us? Only to the extent that (12)'s plural quantifier $\ulcorner \exists \mathbf{x} \urcorner$ is assured of full coverage. It could be, and has been, argued that plural quantifiers are *themselves* vulnerable to the sort of non-standard construal that second-order skeptics have laid so much weight on. It's enough, though, if they are not obviously *in*vulnerable.[10] And it seems clear that they are not. Why should a slight change in wording—'take whatever members of D you like' as opposed to 'take whatever subset of D you like'—be enough to scare the skeptic back to his/her cave?[11]

XII. Parting methodological shot

A certain kind of philosopher is going to react as follows: I see what your non-nominal quantifiers are supposed *not* to be. They are not supposed to be objectual, you've made that clear. Neither are they substitutional, for one can maintain quite consistently that ⌜*x* and *y* are related somehow such that no predicate S of English or any other language is such that things are so related iff they satisfy S⌝. But it's one thing to say that there's a kind of quantifier that doesn't "range over" anything, or signal disjunction/conjunction with respect to a fixed class of substituends; it's another thing to make out in *positive* terms what the alternative is. Until you do that, your theory is just so much wishful thinking and obscurantism.

The accusation rankles, but we don't reject it out of hand. There really *does* seem to be a distinction between, as Quine somewhere puts it, "clarity" and "fluency." And it may well seem that what we have with non-nominals is just fluency. What it really *means* to say that there is something that *x* is and *y* isn't remains desperately unclear; and there is no way to make it clear but through an objectual semantics that reintroduces all the original paraphernalia.

But although there is something to this Quinean objection, let's not lose sight of another point stressed by Quine. Logical formalisms are explained in natural language—what else?—and the best we can hope for is to ground ourselves in a fragment of the language whose logical properties are relatively transparent, or can be made transparent by regimentation and stipulation. Quine was of the opinion that singular objectual quantification was the *only* "generality device" transparent enough not to need explanation in other terms. But that is a separate claim and a debatable one, as we see from the acceptance on their own terms of Boolosian plurals.

Why shouldn't non-nominals be taken on in the same spirit? The question is not supposed to be rhetorical; reasons may exist. It will be hard to know either way until someone clarifies the rules by which a piece of language is judged clear enough to speak for itself.

Notes

[1]The authors would like to thank Vann McGee, Michael Resnik, Michael Glanzberg, Ed Zalta, Richard Heck, Crispin Wright, and especially Gabriel Uzquiano for their (extremely) helpful comments.

[2]Quine 1986. (1st ed, 1970).

[3]Even ignoring those (like Resnik 1988 and Hazen 1993) who disagree already about the monadic case.

[4]The OED gives many examples of such phrases, including on the existential side ‘somehow,’ ‘somewhen,’ ‘somewhere,’ ‘somewhy,’ ‘somewhence,’ ‘somewhither,’ ‘somewise,’ and on the universal side ‘everyhow,’ ‘however,’ ‘everyway,’ ‘everyways,’ ‘everywhen,’ ‘whenever,’ ‘whichever,’ ‘everywhence,’ ‘everywhither,’ ‘anyhow,’ ‘anyway,’ ‘anywhat,’ ‘anywhen, ‘anywhence,’ ‘anywhere,’ ‘anywhither,’ ‘anywise,’ ‘allwhat,’ ‘all-where,’ and ‘allwhither.’

[5]What if these phrases have referential parts? It might be thought that “somewhere” is committal because its substitution-instances (“to New Mexico,” “near Oak Bluffs”), although not themselves referential, usually include place-names. This is one of several reasons why we attach more weight to the other arguments.

[6]A note on the meaning (here) of “is this phrase referential?” The question is not: are there Montague grammarians or other formal semanticists somewhere who have cooked up super-duper semantical values for them, say, functions from worlds to functions from worlds and n-tuples of objects to truth values? The answer to that (which by the way is almost always *yes*) tells us about the commitments of the semanticist, not the commitments of the speaker. Our question is: is the phrase referential in the way that singular terms are, so that someone using the phrase could reasonably be said to be *talking about* its referent, or purporting to talk about its purported referent?

[7]Perhaps a reflexive verb would be better: “*a* comports itself P-ly.”

[8]This is ignoring for now the “null plurality” problem.

[9]Some perspective on the matter of expressive gain: Just by inspection, plural quantification makes for the expressive equivalent of monadic 2nd-order logic. Non-nominal second-order quantifiers taking plural arguments behave like quantifiers over sets of 2nd-order objects = sets of sets of domain elements. The system described in this section thus gives us the expressive equivalent of 3rd-order logic. This is good because 3rd-order sentences are able to “say more” than 2nd-order sentences can. One example is from Kreisel: “a concept that needs a third-order definition is that of measurable cardinal” (1967). (See also Drake 1974, pp. 281-3.) A second example has to do with the 2nd-order consequence relation. Boolos 1985 gives reasons for thinking that an adequate model theory for 2nd-order languages cannot be stated in a 2nd-order language. It is, however, expressible in a 3rd-order language of the sort developed in the text (Rayo and Uzquiano 1999).

[10]Boolos apparently did *not* think that the plural construction was especially resistant to skeptical reinterpretation. (Thanks here to Charles Parsons.)

[11]Let’s assume that the de-nominalizer is as unclear as everyone else how worried to be about the skeptical challenge. Then she has in all likelihood *no idea* whether to see herself as committed to a standard-like semantics. Some might find this troubling, but we think it makes her position stronger. The less the de-nominalization issue has to do with other 2nd-order controversies, the freer we are to decide it on its own terms.

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