IF-THENISM

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Contents

Philosophy is often held to be in crisis. I don't know about that; but certainly it is beset with cris*es*. I call them *hostage* crises because they involve a (relatively) thin, innocent claim and a (relatively) weighty, debatable one; the first is hostage to the second in that the second must hold or the first fails.

A particularly striking form of this is the *paradox*. A certain sentence φ has *got* to be true, we feel. And yet it is hard to see how it *can* be true, given that it implies ψ which is surely false. But the genre extends more widely. Perhaps φ is obvious — it would be ridiculous to doubt it — while ψ is controversial. Perhaps ψ too is obvious, but not *as* obvious as φ. Perhaps the problem is just that ψ is subject to doubts not on the face of it raised by φ. What ties all these cases together is that ψ puts an unexpected new upper bound on φ's credibility, and/or an unexpected new lower bound on the nature of φ's demands.

1. Crises

So, it seems as clear as anything that I am thinking. But, I am thinking only if there is a thinker—a thinking substance. The existence of thinking substances is a metaphysical thesis that is, in the way of metaphysical theses generally, open to doubt.[1] Lichtenberg was famously bothered by this, and found in it a reason to wonder whether it is as obvious that he is thinking as Descartes supposes. ""Thinking is going on" is what one should say, just as one says "Lightning is occurring." Saying "Cogito" is too much, as soon

[1]Hume for instance holds that we find only a bunch of appearances where this substance was supposed to be.

as one translates it as "I am thinking"' (Lichtenberg 1971, para 76, p. 412). What is obvious and indubitable on this view is not *my* thinking but a bout or body of thinking that is filed for understandable reasons under my name—as a certain 1906 Earthquake is filed under "San Francisco" without being especially metaphysically tied to that city. Be all that as it may, $\varphi =$ *I am thinking* does seem more obvious on the face of it than $\psi =$ *There are thinking substances*, and *There are thinking substances* seems to address weightier issues.

This is not the only hostage problem raised by the Cogito. Another comes up on the "thinking" side, even granting the existence of a substance denoted by the first person pronoun. Thinking is a causal notion, and appearances of causal connection are not self-certifying; they may or may not be veridical. If I can be mistaken about the *external* causes of a mental event—these appearances are not caused by a charging rhino if I am dreaming—then why not also the internal causes? My "conclusions" may not be based on my "premises" if I am sleeping, or a swamp-creature, or under a suitable sort of hypnotic spell. Even particular "thoughts" may be illusory, if the predicative bit floats causally free of the subject.[2] The (easy) question of whether I am thinking is thus laid hostage to the (not so easy) question of the causal provenance of what I am pleased to call my thoughts.[3]

Jumping ahead a bit, we find Moore caught up in hostage negotiations with the external world skeptic. The skeptic targets material objects as a class because arguments aimed at particular such objects are not persuasive . I am so sure of having hands that any attempt to undermine that certainty will have to bring in considerations that are more open to question. Considerations that are open to question are hardly in a position to shake my confidence in having hands. And how can I accept the existence of hands while thinking the jury is still out on material objects?

This combination of attitudes may indeed seem preposterous, but Moore would not be pushing the point if the preposterous combination weren't also common. No one, not even Berkeley, doubts we have hands. The existence of material objects strikes him and many other philosophers as a further question that is not so easily settled. The hostage crisis here, far from being defused by the observation that *This is a hand* entails *There are material objects*, depends on that entailment. Why, if φ entails ψ, does ψ appear to raise different and larger issues? Moore thinks the appearance is just mistaken. But this only deepens the mystery of its persistence—especially among logically minded people like ourselves, who ought presumably to know better.

Another potential application of Moore's strategy is to knowledge of the future. *This is a hand* entails, one might think, that it will not prove later to have been a prosthetic fake hand. Philosophers have shied away from this strategy, fearing a modus tollens.

[2] Wright puts the point as follows. Just as I might be dreaming, the causal appearances notwithstanding, I might compatibly with those appearances be *maundering*— where "x is maundering at t just in case x is then in a phenomenologically smooth state which, like dreaming, necessarily precludes the causal conditions for perception *but, in addition, likewise precludes the causal conditions of competent intellection*" (Wright [1991], 106)

[3] "[A] chain of inference is no more a purely phenomenological notion than is, say, remembering how a tune sounded—where, however vivid and accurate the mental impression, and however confident the subject that she thereby recalls the tune, it counts for nothing unless there is an appropriate causal relation between the phenomenological episode and a relevant prior experience of the tune" (Wright [1991], 105).

Knowledge of the present is imperiled, if its content is hostage to hypotheses about the future, rather than knowledge of the future being secured. Somehow then the entailment must be broken, as heroically attempted in passages like the following:

> If we have made sure it's a goldfinch, and a real goldfinch, and then in the future it does something outrageous (explodes, quotes Mrs. Woolf, or what not), we don't say we were wrong to say it was a goldfinch, we don't know what to say... I am not "predicting" in saying it's a real goldfinch, and in a very good sense I can't be proved wrong whatever happens. It seems a serious mistake to suppose that language... is "predictive" in such a way that the future can always prove it wrong. (Austin in Wisdom et al. [1946])

> Not only do I not have to admit that those extraordinary occurrences would be evidence that there is no ink-bottle here; the fact is that I do not admit it. There is nothing whatever that could happen in the next moment or the next year that would by me be called evidence that there is not an ink-bottle here now. No future experience or investigation could prove to me that I am mistaken (Malcolm [1952], 185-6)

Heroics aside, there are possible futures that conflict with, and would falsify, the appearance of an ink-bottle here and now. (A cleaner example: Usain Bolt did not win the Gold Medal after all if technology is developed within six the next six years that turns up banned substances in his refrigerated urine sample.) Why we don't return the kidnapper's calls is just unclear at this point.

Turning from future to past, I am thinking right now about Thales. It would be absurd to question this. It follows from content externalism that I am thinking about Thales (call that φ) only if there *was* such a person as Thales (ψ). If indeed φ entails ψ, we have an obvious truth about the here and now entailing a substantial truth about goings on in 600 BC or thereabouts (Boghossian [1997]). If (as seems plausible) I am thinking about Thales if I'm thinking at all, the fact of my thinking is hostage to something grander and apparently harder to know.

Another class of examples concerns ontology. *Bishops move diagonally* doesn't assert the existence of bishops, but *There are eight pawns on each side* seemingly does assert the existence of pawns. These pawns must presumably be abstract, for the number of concrete pawns is much higher. Whether abstract objects even exist is disputed, but no one seriously doubt there are eight pawns on each side. The number of talking donkeys is clearly 0, and the number of even primes is 1. Whether there exist such things as 0, 1, and 2 — 2 being the reason the number of even primes isn't 0 — however, is endlessly debated. Adapting a famous remark of Carnap's, "[Platonists] give an affirmative answer, [nominalists] a negative one, and the controversy goes on for centuries without ever being solved."

Arithmetical claims are hostage not only to ontology, but also metatheory. Consider Peano's Axioms: 0 is a number, every number has a successor, 0 is not the successor any number, and so on. If one reflects on what the axioms *say*,[4] they are apt to seem clearly true, both separately and together.[5] But now, for the axioms to be true they

[4] And what the induction schema says indirectly, through its instances.

[5] Recall we are bracketing ontological issues.

must be consistent.[6] (Consider the contrapositive: if statements contradict each other, at least one is false.) The consistency of Peano's Axioms ought to be obvious, then, a consequence of arithmetical truisms and platitudes about truth.[7]

And yet the consistency claim tends to be considered unobvious, or at least *less* obvious than the axioms.[8] There are indeed first-class mathematicians who *question* the consistency of first-order Peano Arithmetic (Doyle and Conway [2006]), or believe it has only nonstandard models, which casts doubt on *second*-order Peano Arithmetic.[9] Worries of this sort come up, indeed, in discussions of if-thenism:

> Suppose [the if-thenist] represents sentences φ of arithmetic by means of a material conditional, say, of the form, $PA^2 \supset \varphi$, or some refinement thereof. Suppose also that, in fact, there happen to be no actual ω-sequences, i.e. that the antecedent of these conditionals is false. [...] Then, automatically, the translate of every sentence of the original language is counted as true, and the scheme must be rejected as wildly inaccurate. (Hellman [1989], p. 26).

The translation Hellman prefers is $\Box(PA^2 \supset \varphi)$ conjoined with $\Diamond PA^2$. The box in the first conjunct is because ω-sequences may not exist in *this* world. The diamond in the second is w because they may not exist in any world. The second conjunct would not be necessary if there were not some amount of anxiety about the consistency of arithmetic.

2. Calibration

Here is one way, not the only way, to bring out what is problematic about our mindset in these cases. It is a principle of probability theory that if φ implies ψ, then φ cannot be the more probable of the two. Take for instance the case where φ is $\psi\&\chi$. If ψ is 50% likely, then φ cannot be 90% likely, or even likelier than not.

An popular argument for this runs as follows. If $\pi(P\&Q)$ exceeds $\pi(P)$, then there is another probability assignment π^* that (i) satisfies the stated principle, and (ii) is better calibrated with the truth in this sense: no matter what world we are in, $\pi(X)$ departs further on balance than $\pi^*(X)$ does from X's truth-value.[10] Let's therefore call the problems raised by these examples *calibration* problems. Calibration problems arise when the logically stronger claim seems likelier than its weaker counterpart, that is, when

(1) ψ is at least as probable as φ (since φ entails it)
(2) φ is probable (inspection)
(3) ψ is relatively improbable (inspection)

[6] For Frege, this is how we *establish* consistency: "From the fact that axioms are true it follows that they do not contradict one another." Hilbert thinks Frege has it backwards: "If the arbitrarily posited axioms together with all their consequences do not contradict one another, then they are true and the things defined by these axioms exists. For me, this is the criterion of truth and existence" (Frege and Kluge [1973], p. 12).

[7] See Field [2006] for the reasons this argument isn't available within PA.

[8] One might try to support this as follows. Either Con(PA) is true or it is false. If it is false, then it is not obviously true. If Con(PA) is true, then we have it from Godel that it is not implied by the axioms. (Taking "we have it from Godel" to scope over the whole conditional.)

[9] Feel free to substitute some version of set theory, if you cannot get worked up about the consistency of arithmetic.

[10] By any reasonable measure of on-balance departure.

What have people had to say about this kind of problem? We can distinguish three main lines of response, according to which of the three assumptions is denied:

	denies	maintains
defiance	(1)	the stronger claim is likelier than the weaker
skepticism	(2)	the stronger claim is not as likely as we thought
boosterism	(3)	the weaker claim is likelier than we thought

Table 1. Responses

Of course these are not really incompatible, but since each has us denying something plausible, we should try if possible to align ourselves with one of the three, putting all our bad eggs in one basket.

According to *the defiant ones*, the switched-up probabilities make sense. This might be because we have less *evidence* for the weaker claim,[11] or because the weaker claim is harder to *know*.[12]

Boosters like Moore, Carnap, Alston, and Wright would have us raise our confidence in the weaker claim: the claim that numbers exist, for instance, or that there is a causally unified external world.

Skeptics like van Fraassen and Field would have us lower our confidence in the stronger claim: that the number of faster-than-light particles is 0, for instance, or that lightning causes fire.

If we are not defiant, and want to respect calibration, a further issue arises, namely, why were we initially so confident of φ, or so bothered about ψ? A typical answer on the ψ side is that we let ourselves be rattled by false analogies — expecting numbers to exist in the same way that sofas do, perhaps. A tempting answer, or answer-type, on the φ side is this: we were overly confident of φ because we heard it as making a weaker, or anyway different, claim φ^*, from which ψ did not follow.

This is reminiscent of Kripke's strategy of explaining our assessment of one claim as "really" aimed at another, with which we have the first confused. *Hesperus = Phosphorus* seems contingent because *The morning-visible planet = the evening-visible planet*, which we were running together with it in our minds, really is contingent. *The man drinking a martini is a philosopher* seems true because we are running it together with what the speaker meant in saying by it, which really is true. If we like the "understandable confusion" strategy, we should ask, what is this φ^* whose obviousness is wrongly projected onto φ— wrongly, since φ^* falls short of φ precisely in not implying the controversial ψ?

3. The Defiant Ones

This, the identity of φ^*, is what I want to talk about in the present paper. Not a lot of work has been done on how to cut φ back to something not implying ψ. But the word

[11] Imagine I start out with low evidential probability both for φ and ψ. I learn an ϵ that boosts φ's probability while lowering that of ψ; ψ might be $\neg\epsilon\vee\varphi$. I wind up in a better evidential state where φ is concerned, while my evidence for ψ is degraded, even though φ implies ψ.

[12] As, on a sensitivity theory, it might be easier to know that you have hands than that you (i) have hands or (ii) are at any rate not a handless bran in a vat.

"if" and its cognates loom large in the work that there is. For Vaihinger, author of *Die Philosophie des Als-Ob*,

[VAI] $\varphi^* =$ Everything is as ***if*** φ, apart from ψ

A Husserlian internalist might prefer

[HUS] $\varphi^* = A$, ***if*** we bracket its implication that ψ.

A Lewisian fictionalist will tell you that

[FIC] $\varphi^* = \varphi$ would be the case, ***if*** a certain ψ-implying story were true.

For the figuralist, or advocate of Walton-style prop-oriented make believe,

[FIG] $\varphi^* =$ the X such that ψ-pretenders are, ***if*** X, to pretend that φ.

Presuppositionalists like my occasional self would modify this to

[PSP] $\varphi^* =$ the Y such that ψ-presupposers may, ***if*** Y, assert that φ.

The simplest and prima facie *cleanest* way to strip φ of its implication that ψ, however, is just to *condition* it on ψ, by putting ψ in the antecedent of a conditional whose consequent is φ:

[IFT] $\varphi^* =$ ***If*** ψ, then φ.

The standard complaint about all such proposals is that they involve special pleading and/or wishful thinking—special pleading in that we are reinterpreting only *some* sentences, the ones we don't want to take at face value; wishful thinking in that we interpret them *this* way rather than that, not because the evidence demands it, but because we for philosophical reasons would prefer this interpretation to be correct. I will be arguing that if/thenism, or at least a certain instance of the genre, *may* be at an advantage here with respect to some of its competitors.

Plan of the paper. A recent if-thenist construal of mathematics (pure and applied) is considered in the next section. An irony is noted, that while the conditional may "say less" in the sense that its *demands* are weaker, it says *more* in the sense of visiting its demands on more of the universe. $\psi \rightarrow \varphi$ scores lower tham φ on the scale of logical strength, but higher than φ on the score of aboutness, or subject matter. This extra subject matter prevents $\psi \rightarrow \varphi$ in many cases from serving its intended purposes. The good news is that this problem is greatly mitigated if $\psi \rightarrow \varphi$ is understood to express φ's surplus content relative to ψ. This understanding is at odds, however (this is the bad news) with existing theories of conditionals. Whether this argues against if-thenism, or instead against existing theories of conditionals, is a different question. I will sketch a case for the second alternative at the end.

4. Classic If-Thenism

Hostage-taking raises two sorts of worry. One is hermeneutic: people use φ as though it didn't require ψ. The other is normative: people use φ in a way they're not entitled to, if it does requires ψ. Both concerns are addressed, it seems, by the hypothesis that

φ in practice means *If* ψ*, then* φ—or perhaps *If* χ*, then* φ for some ψ implying χ. For whatever else may be true of *If* ψ*, then* φ, it is not supposed to imply ψ.[13]

So, for instance if a φ which implies the existence of numbers is used in a way that is blind to that implication, we may be tempted to treat φ as short for *If numbers exist, then* φ, or *If numbers had existed,* φ *would have been the case*; or, perhaps, to treat it as short for *If Ax, then* φ, where Ax is an axiomatization of some relevant bit of mathematics. Russell declares in (the first sentence of) Russell [1903] that "Pure Mathematics is the class of all propositions of the form 'p implies q,' where p and q are propositions containing one or more variables, the same in the two propositions, and neither p nor q contains any constants except logical constants." Hilary Putnam agrees, to a point.

> What [Russell] meant was not...that all well formed formulas in mathematics have horseshoe as the main connective, but that mathematicians are in the business of showing that if there is any structure which satisfies such-and-such axioms (e.g. the axioms of group theory [or arithmetic], then that structure satisfies such-and-such further statements (some theorems of group theory or arithmetic). (Putnam [1979], 20).

An if/thenist account of applied mathematics was developed by Terence Horgan in "Science Nominalized" (Horgan [1984]). Horgan's is, in the jargon, a kind of *easy-road* nominalism; the intended contrast is with "hard-road" nominalists such as Hartry Field. Horgan's motivations are both hermeneutic and normative. "The trouble is," he says,

> that many scientific laws .... quantify over various kinds of pure and impure sets, and hence they appear prima facie to be ontologically committed to such entities. How, short of eschewing set-theoretical quantification altogether, might we cancel this apparent commitment? We can do so, I suggest, by regimenting each scientific law statement L into the form
>
> [SNO] Sets exist $> L$
>
> where '$>$' is the counterfactual connective. That is, a regimented law statement will have the form *If there were sets, then it would be the case that L.*

Who is to say that the laws would not be different, though, if the world had mathematical objects in it? Horgan replies that he is interested only in L's implications for what goes on concretely, and that what goes on concretely is counterfactually independent of goings-on in the Platonic realm.

> The non-causal nature of these putative entities means that (i) the very same concreta would exist whether or not sets existed, and (ii) these concreta would behave the very same way whether or not sets existed. Thus, the scientist can describe the lawful behavior of the concreta without actually asserting unregimented laws. He can instead assert nominalized versions, as I shall call them—that is, versions with the counterfactual form ... above. Nominalized laws constitute correct and adequate descriptions of the behavior of concreta in the spatio-temporal, causal nexus, precisely because these concreta would exist and would behave the same

[13] Leaving aside outre' possibilities like: ψ is a logical truth and so implied by everything; or ψ is a Curry-paradoxical sentence like *If* ψ *then* φ; etc.

way whether or not there were sets. We do not lessen the descriptive adequacy of our laws vis-a-vis the behavior of the concreta when we nominalize the laws; rather, all we do is eliminate ontological commitment to sets.

This non-interference claim has been questioned (Baker [2003]). *Causal* independence is one thing, counterfactual independence another. Do we really know the kind of miracle required to bring numbers into being, or what else that miracle might bring in its wake? Horgan's counterfactual reconstruals are *risky*; they depend on an independence assumption that, even if it is true, does not have to be true from the perspective of mathematical physics.

Suppose we read *A projectile's escape velocity from a planet of mass M and diameter 2R is* $\sqrt{2GM/R}$ as "really" saying that $\sqrt{2GM/R}$ is what the escape velocity would be if there were numbers. This leaves room for the (surely unwanted) thought that were there numbers. the velocity required to escape a planet's gravitational field would be higher. Of course the worry strikes us as ridiculous, but that is the point. It would not be ridiculous if the real content of a law-statement were genuinely counterfactual in character. Its ridiculousness is a clue that if-thenists are understanding the conditional another way—as, to anticipate a bit, what *The escape velocity is* $\sqrt{2GM/R}$ adds to the existence of numbers.

5. Misdirection

Suppose as Horgan suggests that $\varphi^* = \pi{>}\varphi$, where π is some appropriate form of platonism, say, platonistic set theory. Then φ^* does not imply the existence of sets. It does however countenance sets in another way, by *talking* about them. Why should this matter?

Remember why we wanted φ^* in the first place. Its job from a hermeneutic perspective is to explain certain prima facie puzzling features of our use of φ. φ^*'s job from a normative perspective is to *vindicate* these features. The claim so far is that φ^* redeems our practice with φ *if* the feature is this: that we use φ in stand-alone contexts as though it didn't imply the existence of sets. But other parts of the practice may need saving as well—how φ behaves when it is embedded in larger contexts, for instance.

Abraham asked the Lord to spare Sodom and Gomorrah for the sake of the righteous men living there, supposing there were enough of them. The haggling started at 50, but Abraham soon bargained the Lord down to 40, 30, 20, and finally 10. Abraham fears that the number of righteous men might not even be 10, but he *hopes* that the number is 10 or more. Or does he? *Abraham hopes #(righteous men)* $\geq$ *10* is in danger of coming out false if Abraham hopes for whatever reason that there aren't any numbers. *Abraham hopes that* φ has, it seems, a better truth-profile if we hear φ (*The number of righteous men is at least 10*) as "really" saying that the number *would* be at least 10 if there were numbers ($\pi{>}\varphi$), for the truth of that counterfactual does not require the existence of numbers.

Any sense of progress here is short-lived, for the problem returns in another guise. The city will not be spared after all, it turns out, if the ten men would *still* be righteous in the presence of numbers—ours is a jealous God who wants the men be driven criminally insane by the existence of immaterial objects beyond himself. Here too we have a claim the practice treats as obvious (*Abraham hopes the number of righteous men is at least*

10) held hostage to an issue (how God feels about *counterfactual* righteousness) that the practice takes no notice of.

Here is an example that you might find less fanciful. Field's case for nominalism in *Science Without Numbers* rests largely on the possibility of finding nominalistic translations of mathematicized physical theories. Field claims as well, though, that the nominalized theory is *better* by ordinary scientific standards than the platonistic original. This is because science aims to give explanations and good explanations are "intrinsic":

> If we need to invoke some real numbers like 6.67 x 10^{11} (the gravitational constant in [SI units]) in our explanation of why the moon follows the path that it does, it isn't because we think that that real number plays a role as a cause of the moon's moving that way...The role it plays is as an entity extrinsic to the process to be explained, an entity related to the process to be explained only by a function (a rather arbitrarily chosen function at that) (Field [1980], 43)

The *real* reason the moon follows that path is to do with the strength of the forces acting on it, not their numerical representation.

Suppose Field is right that explanations should confine themselves to the entities actually doing the work. Presumably then we should *not* say that Sodom and Gomorrah were destroyed because their righteous inhabitants numbered less than 10. The allusion to numbers makes that explanation defective—not to split hairs, *false*—in roughly the way that its allusion to the Oracle casts doubt on *Sodom and Gomorrah were destroyed because the it says in the Book of Genesis that the number of righteous men was less than 10.* Numbers like Books are liable to ruin things in causal contexts, because they are extrinsic to the causal scene.

The if-thenist sees here an opportunity: we can get rid of the numbers—at least of the implication that they exist—by treating φ (*The number of righteous men was less than 10*) as short for *The number would have been less than 10 if there were numbers* ($\pi > \varphi$).

The sense of progress is again short-lived, however. If we don't want to attribute the cities' destruction to the size of an *actual* number—the number of righteous men— we shouldn't want to attribute it either to the size of a *would-be* number: the size a number of that description *would* have had, had there been numbers. Sodom and Gomorrah were destroyed because of an actual fact—the shortage of righteous men—not a conditional one about would-be numbers. It is not just that more men might have been righteous, had there been numbers. What is really doing the causal work is the fact *underlying* the counterfactual, namely, that there weren't enough righteous men.

What does "not enough" even mean, though, in a world without numbers? I've already said that the number of righteous men is too small if it is less than 10. But that is not the question. The question is, how *many* righteous men are too few, not which *numbers* of righteous men are too small.

Well, how many men does it take to bridge the gap between *There are numbers* and *The number of righteous men is less than 10*? A bit more precisely, which of the following hypotheses

(1) There are no righteous men in Sodom and Gomorrah.
(2) There is at most one such man.
(3) There are at most two such men.

(4) ....
(5) ....
....

do we need to complete the following enthymeme:

> There are numbers
> ....
> The number of righteous men < 10.

The missing premise is of course that are at most nine righteous men. A key fact about *all* these bridge-gappers (or missing premises, or whatever we call them) is that they are not at all about numbers. They are caught between two numerical bookends, but that is not the same thing.[14] The only kind of if-thenism that has a chance is the kind that treats *If* ψ *then* φ as expressing whatever it is that bridges the gap between ψ and ϕ.

6. Concrete Accuracy

Horgan interestingly enough winds up with a view in this neighborhood as well, though he takes a quite different route. Rather than rely on (nominalistically disreputable) possible worlds, he adopts a Goodmanian account of counterfactuals: $\pi{>}\varphi$ is true just if truths cotenable with π combine with it to imply φ, or what comes to the same, truths cotenable with π imply $\pi \supset \varphi$. π in the case at hand is just Platonism. $\pi{>}\varphi$ is true, then, just if Platonism-friendly truths imply $\pi \supset \varphi$. The *main* cotenable truth one needs for this purpose, he thinks, is $[c]\varphi$, glossed as *It is concretely accurate that* φ. φ is concretely accurate, more or less, if it is false (should it be false) solely because of the non-existence of mathematical objects.[15]

How does this line up with the "missing premise" view in the last section? Suppose φ says that the number of planets is 8. Then we want to see how $[c]\varphi$ compares to the missing premise in

> π (Platonism)
> .....
> φ (The number of planets = 8.)

[14]Unless one wants to say that *There are dogs* is covertly about numbers, because it is caught between *There are numbers* and *The number of dogs* $\geq$ *1.*

[15]φ* in his view can be equated either with $\pi{>}\varphi$ or $[c]\varphi$:

> the notion of concrete accuracy ... provides us with an alternative way to nominalize the quantitative sentences of science. We can regiment a quantitative sentence φ metalinguistically, into the form *'φ' is concretely accurate.* Or, better, we can remain in the object language by introducing an operator '[c]', to be read "It is concretely accurate that...." The semantics for this operator will be given in terms of our semantical predicate of sentences: $[c]\varphi$ is true iff φ is concretely accurate. [Compare Rosen on nominalistic content in Rosen [1992].]....[U]nder a reasonable account of cotenability the two alternative modes of nominalizing should turn out equivalent, relative to the scientific sentences that interest us. That is, for any quantitative sentence φ of science, it should turn out that $[c]\varphi$ is true iff $\pi{>}\varphi$ is true. Also, the counterfactual formulation has the advantage of using a familiar ... connective from ordinary language. For these reasons I shall continue to let my official proposed regimentation be the counterfactual one. But the alternative version would do as well... (ibid, 544)

Suppose first that φ is true: the number of planets is 8. Horgan's rule tells us that $[c]\varphi$ will then be true as well. A truth, after all, is not false for *any* reason, in particular not because π is false; indeed π is *true* since φ implies it. The missing premise too, one might think, will have to be true in that condition; for if *The number of planets = 8* is true, then what it adds to Platonism (given that the added bit is implied by φ) will have to be true as well.

Suppose next that φ is false. Then $[c]\varphi$ is true or false, we are told, according to whether φ is false just because the numbers are missing. For *The number of planets = 8* to be false just because the numbers are missing would seem to mean something like this: a fact obtains that takes us from the existence to *The number of planets = 8*; so the fault lies entirely with π. This is also the condition under which the missing premise is true, for the job of an enthymeme-completer is precisely to take us from π to φ. For φ to be false *not* just because the numbers are missing means something like this: a fact obtains that falls short of *The number of planets ≠ 8* by the existence of numbers—that there aren't any planets, for instance, or that there is exactly one planet, or etc.

7. Remainders

Some rather mysterious phrases have been cropping up in the last few pages: a hypothesis "bridging the gap" between ψ and φ; or summing up "what φ adds to ψ"; or "falling short of φ precisely by ψ." Wittgenstein speaks in a similar vein of the *remainder* when ψ is subtracted from φ

> what is left over if I subtract the fact that my arm goes up from the fact that I raise my arm? (PI, par 621).

Goodman's notion of *surplus content* belongs here too. A black raven Rudy confirms *All ravens are black* inductively by virtue of confirming "the rest of" φ, namely, *All ravens other than Rudy are black.* A silver coin in my pocket confirms *All the coins in my pocket are silver*, but not inductively since the unexamined coins are no likelier silver than before.

All these phrases seem to be pointing in the same direction—toward a non-truth-functional connective $\varphi{\sim}\psi$ whereby φ's implication ψ is cancelled or waived or stripped away.[16][17] I am not sure there is a perfectly colloquial way to express $\varphi{\sim}\psi$ in English, but the following are in the ballpark: *φ, except maybe not ψ*; *φ, ignoring the bit about ψ*, and *φ, but possibly for ψ*. (I'll propose later that *If ψ, then φ* can be used to express remainders as well.) So, for instance,

(1) Every Justice spoke up, with the possible exception of Thomas.
(2) Kennedy was killed by someone other than Oswald, or indeed by Oswald.
(3) The triangles are congruent, except maybe in not being the same size.
(4) Pete won, ignoring the possibility that he folded.

What do these statements say? How is the proposition $\varphi{\sim}\psi$ to be identified on the basis of φ and ψ? The obvious analogy is with subtraction in arithmetic. The corresponding problem there is that of identifying m minus n on the basis of m and n. This is solved,

[16] The operator still applies if φ does not imply ψ, only its action cannot be described in this case as cancelling an implication.

[17] I like to think of $\sim$ as undoing the effect of conjunction, or better, as the operator such that conjunction undoes *its* effect: $(\varphi{\sim}\psi)\&\psi$ is true in the same worlds as φ.

we know, by looking for the unique number ρ such that adding ρ to n gets you back to m. An analogous proposal about $\varphi{\sim}\psi$ would be this: it's the ρ that has to be added to ψ to get φ.

The problem is that there might be any number of ρs such that ψ and ρ imply φ, ranging from $\psi\supset\varphi$ to $\psi\equiv\varphi$ to φ itself. Some of these ρ's are, once again, better than others. The ρ we want, it seems, is the hypothesis that completes the enthymeme ψ, ..., $\therefore\ \varphi$ in the maximally ψ-beholden way. How on this way of doing things are we to find *Every Justice spoke up*$\sim$*Justice Thomas spoke up*? First let's set out the relevant enthymeme.

Thomas spoke up.
....
Every Justice spoke up.

To complete this in a maximally ψ-beholden way, we look for the feature of certain *Thomas spoke up*-worlds whereby they are *Every Justice spoke up*-worlds. That feature is, I take it, that every other Justice spoke up in them. Again, what is the shortest path from *Oswald didn't shoot Kennedy* to *Someone other than Oswald killed Kennedy*? What is going on in certain *Oswald didn't do it*-worlds to make them *Someone else did it*-worlds? Someone shot Kennedy in those worlds. *Someone else shot Kennedy*$\sim$*Oswald didn't shoot him* is therefore *Someone shot Kennedy.* Let's try finally to strip *Pete won* of the implication that Pete called. What is going on in certain *Pete called*-worlds to make them *Pete won*-worlds? Well, Pete had the better hand in those *Pete called*-worlds. *Pete won*$\sim$*Pete called* is therefore *Pete had the better hand.*[18][19]

8. Affinities

The only viable form of if-thenism, we said, is the kind that has *If* ψ *then* φ expressing $\phi{\sim}\psi$ (= the remainder when ψ is subtracted from φ). If that is right, then if-thenism is not viable in *any* form unless $\varphi{\sim}\psi$ is a possible reading of *If* ψ *then* φ. *Is* it? This is what I want to explore in this section, and then again at the end.

Remainders and conditionals do seem to turn up in a lot of the same places. Horgan's form of if-thenism is a case in point. *Numbers exist*$>$*The number of stars is finite* stands or falls in his view with the concrete content of *The number of stars is finite.* The concrete content of *The number of stars is finite* is that there are only finitely many stars. For there to be finitely many stars is what *The number of stars is finite* adds to *Numbers exist. Numbers exist*$>$*The number of stars is finite* thus stands or falls with *The number of stars is finite*$\sim$*Numbers exist.*

Another reason not to insist on a sharp distinction is that remainders are apt to be *formulated* in conditional terms without anyone noticing or remarking on the fact. Recall our examples from above—*Every Justice with the possible exception of Thomas spoke up, These triangles are congruent, waiving the same-size requirement.* The corresponding conditionals are

(1) If Thomas spoke up, then all the Justices spoke up.
(2) If Oswald didn't shoot Kennedy, then someone else did.

[18] A more precise statement, in terms of truthmakers, is given in section 9.

[19] Precis of next section: Remainders $\varphi{\sim}\psi$ and conditionals have a number of affinities, ranging from the language used to express them to the graphical representations that suggest themselves when we try to calculate their truth-conditions.

(3) If these triangles are the same size, they're congruent.
(4) If Pete called, he won.

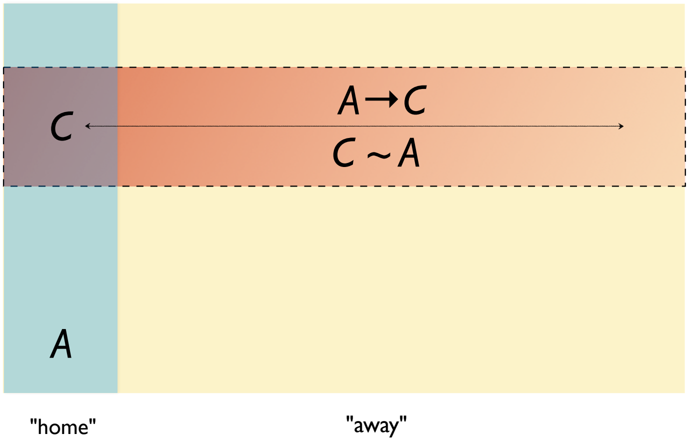
These seem in each case to admit an interpretation whereby they stand or fall with $\varphi \sim \psi$. The Oswald conditional is decided by whether Kennedy was killed. The Thomas conditional is decided by whether everyone other than Thomas spoke up. The Pete conditional is decided by whether Pete had the better hand. That all these conditionals can be read incrementally is a second point of analogy between remainders and conditionals.

A third is to do with the phenomenon of non-catastrophic presupposition failure. The falsity of its presupposition π may *sometimes* make a sentence φ unevaluable—as *The king of France is bald* seems unevaluable on account of France's lack of a king. But there are other failed-presupposition sentences that strike us as just plain false: *The king of France is bald and pigs fly*, for instance, and *The king of France is sitting in this chair.* φ will strike us as false, despite the failure of its presupposition π, if φ adds something to π—*Pigs fly*, *The chair is not empty*—which is independently evaluable and turns out to be false, that is, if $\varphi \sim \pi$ is false. $\varphi \sim \pi$ is a remainder, obviously, but one is strongly tempted to formulate it as a conditional (Lasersohn [1993]): Even if France has a king, it is not the case that: He is bald & pigs fly; Even if France has a king, still, he is not sitting in this chair.

The temptation persists even when the conditional formulation is risky. Imagine that someone convinces us, never mind how, that the king of France is sitting undetectably in that chair, if he exists (he is a master illusionist, the chair is really a throne, etc.). If *The king of France is sitting in this chair* seemed false before, it continues to now, based on the categorical fact that the chair is empty. Whether it *remains* empty on the hypothesis that France has a king seems just irrelevant. The reformulated test is tempting only insofar as we are reading *If* π *then* φ incrementally, as expressing φ's surplus content with respect to π. Since it undoubtedly *is* tempting, that must be how we are reading it.

A fourth point of analogy between *If* ψ *then* φ ($\psi \rightarrow \varphi$) and φ *but possibly for* ψ ($\varphi \sim \psi$) is that they respect modus ponens and centering. They imply $\psi \supset \varphi$, that is to say, and are implied by $\varphi \& \psi$.

A fifth point of analogy between $\psi \rightarrow \varphi$ and $\varphi \sim \psi$ is that they lend themselves to a similar sort of graphical representation. Both require us to extrapolate in some way from the distribution of φ in "home"-worlds (ψ-worlds) to its likely distribution in "away"-worlds ($\neg\psi$-worlds). $\psi \rightarrow \varphi$ and $\varphi \sim \psi$ agree with φ, and each other, in the home-region. The challenge with conditionals and remainders alike is to find a way to model their away behavior on their behavior at home.



9. Home and Away

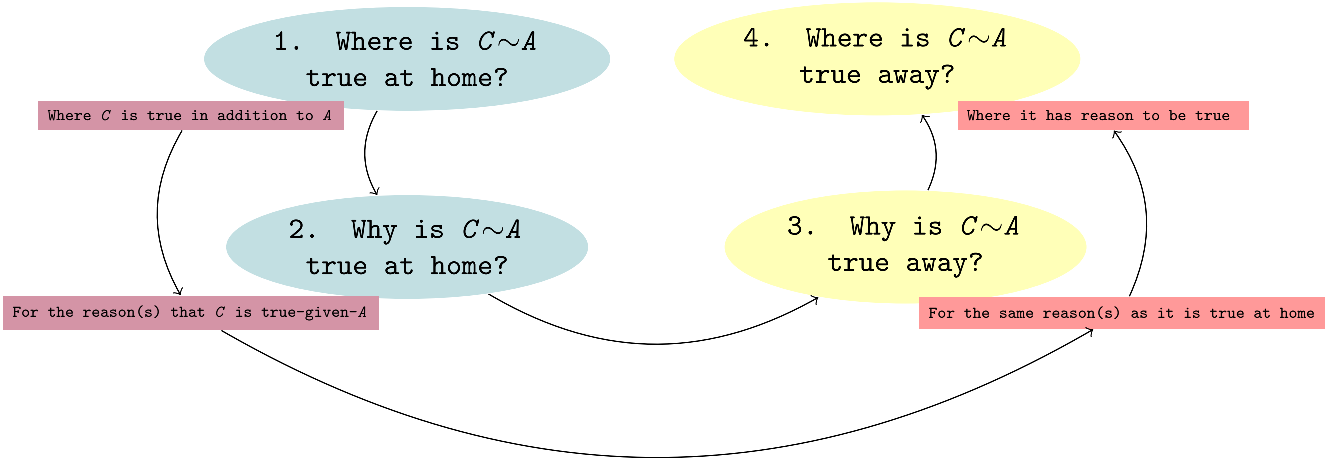
How in graphical terms are we to solve "$C{\sim}A = R$" for R?[20]

R informally speaking is the result of extending C's behavior at home, where A holds, to the away region, where A is false. From this it seems that R's truth-value in the away region ought to be controlled by the same sorts of factors as distinguish $A\&C$-worlds from $A\&\neg C$ worlds.

Looking back at the home vs away diagram, we can see that extrapolation is going to require two types of rule. "Home" rules speak to R's behavior at home, that is, within the A-region. An R that continues C beyond that region should at a minimum follow C's lead within it. "Away" rules speak to R's behavior *outside* the A-region. If R is to divide up away-worlds on the same principle as at home, its away behavior should agree in some still undetermined way with its behavior at home.

A second, cross-cutting, distinction is between "classifying" conditions, which are to do with *whether* R is true in a given world, and "rationalizing" conditions are to do with *how and why* R is true in a world. This gives us four types of condition overall—home-classifying, home-rationalizing, away-classifying, and away-rationalizing. The following order seems logical: 1. HC, 2. HR, 3. AR, 4. AC. Here in broad outline is how the construction is going to go:

[20]The extrapolation diagram assumes that C implies A, and I sometimes talk this way for simplicity, suggesting for instance that $C{\sim}A$ strips C of its implication that A. I hope this does not cause confusion. The definition in the main text of $C{\sim}A$, though, allows C and A to be independent. Take for instance *If that guy's your brother, your brother's an idiot.* Obviously *Your brother's an idiot* does not imply *That guy's your brother.* That does not prevent us from reading it incrementally, as turning on *Your brother's an idiot*'s surplus content over *That guy's your brother.* The surplus content on our definition is *That guy's an idiot.*



So, let's do it, beginning at home and extending patterns established there into the away-region. R should have the same truth-value as C at home. It should be true if $C\&A$, and false if $\neg C\&A$. Since it asks R is to agree truth-value-wise with C, we call this

Agreement (1)

$$R \text{ is } \begin{Bmatrix}\text{true}\\ \text{false}\end{Bmatrix} \text{ in a home-world } w \text{ just if } C \text{ is } \begin{Bmatrix}\text{true}\\ \text{false}\end{Bmatrix} \text{ in } w.$$

Next, given that a home-world w is R, how does it acquire that status? To answer a question with a question, why is w on the $A\&C$ side of the line rather than the $A\&\overline{C}$ side? (Why is it a *The # of dragons = 0* world rather than one where *The # of dragons ≠ 0*)? *That* is why R holds true in w. R's reason for being true (false) in a home-world w is whatever makes C true (false) in w, *given that A is true there.*[21]

Reasons (2)

$$R \text{ is } \begin{Bmatrix}\text{true}\\ \text{false}\end{Bmatrix} \text{ in home-world } w \text{ for the reason(s) } C \text{ is } \begin{Bmatrix}\text{true}\\ \text{false}\end{Bmatrix} \text{ in } w \textit{ given } A.^{22}$$

Next we have to specify R's ways of being true/false in *away* worlds. A hypothesis "goes on in the same way" from the home-region if its truth is controlled by the same factors. R should not acquire new truthmakers and falsemakers when it leaves home.

Integrity (3)

[21] In this case: It doesn't have any dragons.

[22] **Reasons** could stand to be clarified. What are C's reasons for being true *given* A, in an A-world? A reason for C to be true-given-A is a truthmaker X for $A \supset C$ that is consistent with A and makes the fullest possible use of A. X does that *if it minimizes the extent to which $B \supset C$ is also implied.* for B weaker than A. The official definition is in three steps. A^- in the first step ranges over hypotheses weaker than—asymmetrically implied by—A. (Alternatively we could let it range over A's parts; this paper is not about content-parts, however, so we use the definition given.

(1) X' *uses more of* A than X if $\{A^- \mid A^-, X' \nvDash C\} \subsetneq \{A^- \mid A^-, X \nvDash C\}$
(2) X is *wasteful in* w if a X' holding in w uses more of A than X does.
(3) X is *wasteful* simpliciter if it is wasteful in every A-world where it obtains.

$$R \text{ is } \begin{Bmatrix}\text{true}\\ \text{false}\end{Bmatrix} \text{ in away-worlds for the same reasons as it was } \begin{Bmatrix}\text{true}\\ \text{false}\end{Bmatrix} \text{ at home.}$$

R's truth-value(s) in away-worlds is a function, finally, of the available *reasons* for R to be true/false in such worlds.

Projection (4)

$$R \text{ is } \begin{Bmatrix}\text{true}\\ \text{false}\end{Bmatrix} \text{ in an away-world } w \text{ just if it has reason to be } \begin{Bmatrix}\text{true}\\ \text{false}\end{Bmatrix} \text{ in } w.$$

More simply: $C{\sim}A$ is true (false) just where $A{\supset}C$ ($A{\supset}\neg C$) has a targeted truthmaker: a truthmaker consistent with A (by ***Integrity***) and making optimal use of A (by ***Reasons***, as elaborated in the footnote).[23]

10. Exceptionalism

Near the beginning, in section 3, we looked at a number of ways of cutting φ back to a φ^* that did not imply ψ. Among the candidates for the φ^* role were *Everything is as if* φ*,* ψ *aside*; φ, bracketing its implication that ψ; φ *would be the case if a certain* ψ*-entailing story were true*; X *obtains*—the condition under which ψ-imaginers are to imagine that φ; Y *obtains*—the condition under which ψ-presupposers are entitled to suppose that φ. To these we added *If* ψ*, then* φ and then, as our preferred interpretation of that conditional, $\varphi{\sim}\psi$, or φ *but possibly for* ψ.

Once again, a standard worry about such proposals is that they involve special pleading and/or wishful thinking—special pleading in that we are reinterpreting *these* sentences rather those without exactly saying what justifies the different treatment; wishful thinking in that we interpret them *this* way rather than that, not on the basis of evidence, but because of the advantages it confers. The problem actually arises twice for us, in connection first with the conditional $\psi{\rightarrow}\varphi$ that we use in our theory, and then the ordinary claim φ whose real content is supposedly given by that conditional. I will be focussing on the first question. What is there to suggest that $\psi{\rightarrow}\varphi$ has an incremental reading, on which it stands or falls with $\varphi{\sim}\psi$?[24]

The fact that it would suit our philosophical purposes is no reason to think that $\psi{\rightarrow}\varphi$ can be read incrementally, in terms of φ's surplus content with respect to ψ. About this the objector is right. But, that is not the only reason to think it can be read that way (Yablo [2016]). We can find examples from far afield where the incremental reading seems clearly available. To bring this out, I will try to force the incremental reading by grouping the examples into pairs $A{\rightarrow}C$ and $A{\rightarrow}D$, where C and D are equivalent in the A-region.

Why would this help? Standard theories treat $A{\rightarrow}C$ and $A{\rightarrow}D$ as equivalent when C and D coincide on A; the A-region is all that matters where they're concerned. The

[23] Another route to $C{\sim}A$ brings subject matter in more explicitly. We can define, for each statement S and subject matter $\mathtt{m}$, the part or portion of S that is not at all about $\mathtt{m}$. $C{\sim}A$ then becomes the portion of C that is not at all about $\mathtt{whether}\ A$. Yablo [2012] has details.

[24] This was discussed already in the section on Affinities, but there is more to be said. The second—what is there to suggest that φ can have $\varphi{\sim}\psi$ as its real content?—is discussed in the last few chapters ofYablo [2014].

conditional probability of C on A takes no notice, for instance, of C's behavior in $\neg A$-worlds; it looks just at the proportion of A-worlds that are C. Likewise for closest-world theories. If the same A worlds are C as D, then the closest A-world is C just in case it is D.

What this means in practice is that standard theories cannot distinguish, say, $a{=}b{\rightarrow}Fa$ from $a{=}b{\rightarrow}Fb$—for Fa is true in the same $a{=}b$-worlds as Fb. Incremental conditionals can and do distinguish these. The gap between $a{=}b$ and Fa is filled by Fb; the gap between $a{=}b$ and Fb is filled by Fa. Insofar as $a{=}b{\rightarrow}Fa$ appears to "say" that Fb, and vice versa, we are reading the conditionals incrementally.

See the table for six examples where $A{\rightarrow}C$ and $A{\rightarrow}D$ seem true and false respectively, when and because C's surplus content over A is true while D's is false.

A bit of commentary will be helpful. The first, Donnellan-style, example, ought to be familiar. *If a=b, then Fa* says, if we make most of the antecedent, that Fb, while *If a=b, then Fb* says that Fa. Fa can come apart truth-value-wise from Fb in worlds where a isn't b.

For the second, it is agreed, let us say, that brain states necessitate mental states. But there is controversy about which necessitate which. Our currently best psychophysical theory, theory T, says that Z-fiber firings necessitate anxious feelings. I have no idea what my brain state was at noon, but I take the theory's word for it that *If my Z-fibers were firing at noon, I felt anxious then.* I am aware of course that I have other ways of determining whether I was anxious at noon, like trying to remember. Suppose I determine that I *was* anxious. Then the theory was right about the mental state corresponding to Z-fiber firings, if mine were firing at noon. Suppose on the other hand that I was not anxious. Then the theory was wrong if mine were firing at noon. Still convinced of the theory, I in fact recall I was *not* anxious, whence my Z-fibers were probably not firing at noon. Still this conditional seems true: *If they were firing at noon, then the theory was wrong about them.* While this one still seems true as well: *If they were firing at noon, then I was feeling anxious at noon.* In worlds where they were firing, however, the theory is right iff I was anxious, because that's what the theory says.

The third has an analogous structure to the first. *If a agrees resembles b, then b is like so*, turns on whether a is like so; mutatis mutandis for *If a resembles b, then a is like so.* In worlds where a and b do not in the relevant respect resemble each other, there is no reason that *a is like so* cannot be true while *b is like so* is false.

What you need to know for the fourth is that according to a certain book, nothing ever changes. *If the book is correct, nothing changes* is true, because that by hypothesis is what the book says. *If the book is correct, it says that nothing ever changes* is false; to be correct, it should say that things change (since they do). The consequents are equivalent in worlds where the book is correct.

The fifth is fairly self-evident. The gap between *Called* and *Won* is filled by *Better hand.* The gap between *Called* and *Better hand* is filled by *Pete knew whose hand was better.* Clearly in a world where Pete doesn't call, he might have the better hand but not realize it.

For the sixth, imagine a Knute Rockne movie where the coach is never satisfied; you say that was your best, and he demands *better* than your best. Wanting better than that is the same as wanting better than your best in worlds where that's your best. But the first, superficially contradictory, conditional is true; it says the coach wants better than *that.* While the second, superficially coherent conditional, is false; it says the coach wants better than x, where x really is your best.

Antecedents are identical		Consequents agree at home	Conditionals disagree away from home
A. That guy is Smith's murderer	$\rightarrow$	C. Smith's murderer is insane. D. That guy is insane.	TRUE cuz: That guy is insane FALSE cuz: Smith's murderer is NOT insane.
A. My Z-fibers were firing,	$\rightarrow$	C. I was anxious. D. The theory is right about Z-fibers	TRUE cuz: The theory says so. FALSE cuz: I was not feeling anxious.
A. Bizet and Verdi are the same height,	$\rightarrow$	C. Verdi is short. D. Bizet is short.	TRUE cuz: Bizet is short. FALSE cuz: Verdi is tall.
A. This book is correct	$\rightarrow$	C. Nothing ever changes D. It says nothing ever changes..	TRUE cuz: That's what the book says ... FALSE cuz: Things change.
A. Pete called,	$\rightarrow$	C. He won. D. His hand was better.	TRUE cuz: Pete had the better hand. FALSE cuz: Pete didn't know Mr Stone's hand.
A. That's your best,	$\rightarrow$	C. I want better than your best D. I want better than that	TRUE cuz: I want better than that FALSE cuz: I don't want the impossible.

TABLE 2. Incremental conditionals; C and D agree in A-worlds but $A\rightarrow C$ is true and $A\rightarrow D$ is false

11. Conclusion

If-thenism is at its best, I have argued, if $\psi\rightarrow\varphi$ is taken to mean whatever it is that $\varphi\sim\psi$ means. Whether its best is good enough is hard to say.

That will depend on, among other things, how much of interest is left when ψ is subtracted from φ.[25] The prospects for subtracting Peano Arithmetic's consistency from its truth seem fairly dim. The prospects for subtracting Thales from thoughts about Thales are a bit better, and for pulling φ's "predictive" content out of its total content, are better yet. Skimming off the abstract ontology is easier if φ has concrete content, but not impossible, dare I hope, if φ is, say, the intermediate value theorem. These are tricky issues, to be sure, but they are also partly technical issues, and so amenable to technical methods. Where there are methods, there is hope. Calculemus!

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[25]See Yablo [2014] on degrees of extricability.

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